The Validation and Falsificacion of Chiral Nuclear Forces

E. Ruiz Arriola

University of Granada
Atomic, Molecular and Nuclear Physics Department

, YORK (UK)

November 7th 2017

with Rodrigo Navarro Pérez, José Enrique Amaro Soriano

References

- [1] **Coarse graining Nuclear Interactions**
  Prog. Part. Nucl. Phys. **67** (2012) 359
- [2] **Error estimates on Nuclear Binding Energies from Nucleon-Nucleon uncertainties**
  arXiv:1202.6624 [nucl-th].
- [3] **Phenomenological High Precision Neutron-Proton Delta-Shell Potential**
  Phys.Lett. B724 (2013) 138-143.
- [4] **Nuclear Binding Energies and NN uncertainties**
  PoS QNP **2012** (2012) 145
- [5] **Effective interactions in the delta-shells potential**
  Few Body Syst. 54 (2013) 1487.
- [6] **Nucleon-Nucleon Chiral Two Pion Exchange potential vs Coarse grained interactions**
  PoS CD12 (2013) 104.
- [7] **Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production**
  Phys.Rev. C88 (2013) 024002, Phys.Rev. C88 (2013) 6, 069902.
- [8] **Coarse-grained potential analysis of neutron-proton and proton-proton scattering below the pion production threshold**
  Phys.Rev. C88 (2013) 6, 064002, Phys.Rev. C91 (2015) 2, 029901.
- [9] **Coarse grained NN potential with Chiral Two Pion Exchange**
  Phys.Rev. C89 (2014) 2, 024004.
- [10] **Error Analysis of Nuclear Matrix Elements** Few Body Syst. 55 (2014) 977-981.
- [11] **Partial Wave Analysis of Chiral NN Interactions** Few Body Syst. 55 (2014) 983-987.

- [12] **Statistical error analysis for phenomenological nucleon-nucleon potentials** Phys.Rev. C89 (2014) 6, 064006.
- [13] **Error analysis of nuclear forces and effective interactions**J.P.G42(2015)3,034013.
- 14] **Bootstrapping the statistical uncertainties of NN scattering data** Phys.Lett. B738 (2014) 155-159.
- [15] **Triton binding energy with realistic statistical uncertainties** (with E. Garrido) Phys.Rev. C90 (2014) 4, 047001.
- [16] **The Low energy structure of the Nucleon-Nucleon interaction: Statistical vs Systematic Uncertainties** arXiv:1410.8097 [nucl-th] (Jour. Phys. G, in press).
- [17] **Low energy chiral two pion exchange potential with statistical uncertainties** Phys.Rev. C91 (2015) 5, 054002.
- [18] **Minimally nonlocal nucleon-nucleon potentials with chiral two-pion exchange including Δ resonances** (with M. Piarulli, L. Girlanda, R. Schiavilla). Phys.Rev. C91 (2015) 2, 024003.
- [19] **The Falsification of Nuclear Forces** EPJ Web Conf. 113 (2016) 04021
- [20] **Statistical error propagation in ab initio no-core full configuration calculations of light nuclei** (with P. Maris, J.P. Vary) Phys.Rev. C92 (2015) no.6, 064003
- [21] **Uncertainty quantification of effective nuclear interactions** Int.J.Mod.Phys. E25 (2016) no.05, 1641009
- [22] **Binding in light nuclei: Statistical NN uncertainties vs Computational accuracy** (with A. Nogga) arXiv:1604.00968
- [23] **Precise Determination of Charge Dependent Pion-Nucleon-Nucleon Coupling Constants** arXiv:1606.00592
- [24] **Three pion nucleon coupling constants** Mod.Phys.Lett.A,Vol.31,No.28(2016)1630027

INTRODUCTION

Error Analysis and Nuclear Structure

- What is the predictive power of theoretical nuclear physics ?

INPUT from Experiment $\to$ CALCULATION $\to$ OUTPUT vs Experiment

- Experiment much more precise than theory, but how much ?

$$\Delta M^{\rm exp} < 1{\rm KeV} \ll \Delta M^{\rm th} =?$$

- Theoretical Predictive Power Flow: From light to heavy nuclei

$$H(A) = T + V_{2N} + V_{3N} + V_{4N} + \cdots \to E_2, E_3, E_4, \dots$$

- Chiral expansion allows to compute $V_{2N}, V_{3N}, V_{4N} \dots$ systematically so that one has the hierarchy

$$V_{2N} \gg V_{3N} \gg V_{4N} \gg \dots$$

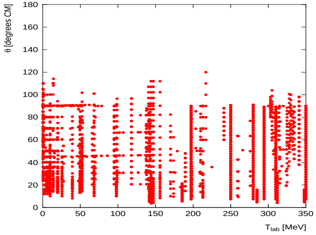
- Forces depend on the renormalization scale allowing to adjust $B_2, B_3, \dots$

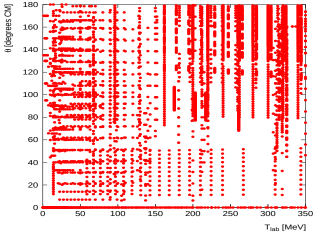
Sources of uncertainties/bias

- Input data from experiment
- Choose the form of the potential
- How to estimate theoretical errors based on INPUT data

$$INPUT = NN, 3N, \cdots \to OUTPUT = 4N, \ldots$$

- First Step: INPUT=NN scattering data
- OUTPUT=NN scattering amplitudes





NN-OnLine http://nn-online.org 7 June 2013

NN-OnLine http://nn-online.org 7 June 2013

The issue of predictive power in Chiral Approach

- Chiral forces are UNIVERSAL at long distances

$$V^{\chi}(r) = V^{\pi}(r) + V^{2\pi}(r) + V^{3\pi}(r) + \dots \qquad r \gg r_c$$

- Chiral forces are SINGULAR at short distances

$$V^{\chi}(r) = \frac{a_1}{f_\pi^2 r^3} + \frac{a_2}{f_\pi^4 r^5} + \frac{a_3}{f_\pi^6 r^7} + \dots \qquad r \ll r_c$$

- They trade model independence for regulator dependence
- What is the best theoretical accuracy we can get within "reasonable" cut-offs ?
- What is a reasonable cut-off ?

$$r_c = ?$$

Bottomline

THE PROBLEM

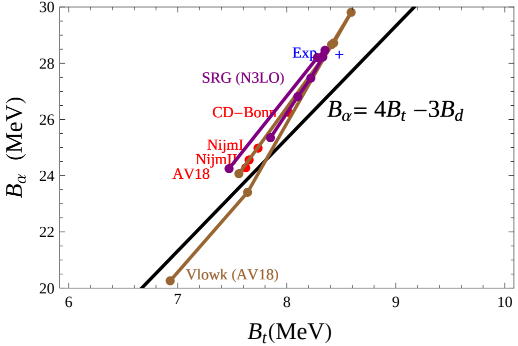
- GOAL: Estimate uncertainties from IGNORANCE of NN,3N,4N interaction
  Reduce computational cost
- Statistical Uncertainties: NN,3N,4N Data
  Data abundance bias
- Systematic Uncertainties: NN,3N,4N potential
  Many forms of potentials possible
- Confidence level of Imperfect theories vs Perfect experiments

OUR APPROACH

- Start with NN
- Fit data WITH ERRORS with a simple interaction
- Compare different interactions
- Estimate uncertainties of Effective Interactions and Matrix elements
- Propagate errors to A=3,4, etc.

Tjon-Line Correlation

- α-triton calculations find a linear correlation between



- SRG argument with on-shell nuclear forces

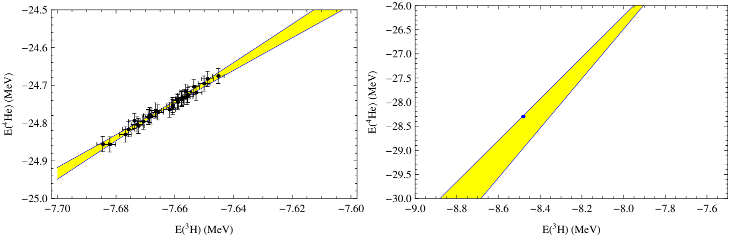
Timoteo, Szpigel, ERA, Few Body 2013

$$B_\alpha = 4B_t - 3B_d \quad \to \quad \frac{\partial B_\alpha}{\partial B_t}\Big|_{B_d} = 4$$

Tjon-Lines: numerical accuracy

(with A. Nogga)

$$\Delta E^{\rm stat}_{\rm triton} = 15{\rm KeV} \qquad \Delta E^{\rm stat}_{\alpha} = 50{\rm KeV}$$



- 4-Body forces are masked by numerical noise in the 3 and 4 body calculation if

$$\Delta^{\rm num}_t > 1{\rm KeV} \qquad \Delta^{\rm num}_t > 20{\rm KeV}$$

The question and its consequences

- We have N DATA with UNCERTAINTIES

$$O_1 \pm \Delta O_1 \quad \dots \quad O_N \pm \Delta O_N$$

- We have a theory depending on M-PARAMETERS

$$O_1(p_1, \dots, p_M) \quad \dots \quad O_N(p_1, \dots, p_M)$$

- Does theory EXPLAIN data ?
  YES (Validate) NO (Falsify)
- Statistical Answer:
  If uncertainties are a gaussian distribution

$$O_i^{\rm exp} = O_i^{\rm th} + \xi_i \Delta O_i$$

Define the LEAST SQUARES SUM

$$\chi^2_{\rm min} \equiv {\rm min}_{p_1, \dots p_M} \sum_{i=1}^{N} \left[\frac{O_i(p_1, \dots p_M) - O_i^{\rm exp}}{\Delta O_i} \right]^2$$

The probability p that the theory explains the DATA is $> 68\%$ if

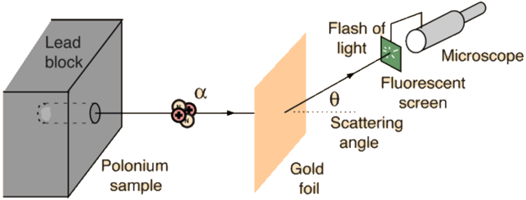
$$\frac{\chi^2_{\rm min}}{\nu} = 1 \pm \sqrt{\frac{2}{\nu}} \qquad \nu = N - M \quad {\rm d.o.f \ (degrees\ of\ freedom)}$$

Theory versus experiment

- A good theory can tell us what experiments are wrong
- A good experiment can tell us what theories are wrong
  1. Assume theory AND experiment to be correct
  2. If we find no contradiction we validate theory and experiment
  3. Experiment , finite number of data, finite precision
  4. Theory , approximations
- Important questions
  1. Does QCD describe hadronic interactions ?
  2. Does ChPT describe low energy hadronic interactions ?
- Confidence level (statistics)
  Example: **AB scattering is described by scheme S with 68 percent confidence**

SCATTERING

Scattering experiment



- Scattering experiments measure FORCES
- Counting rates

$$\frac{R^2 N_{\text{out}}(\theta,\phi)}{N_{\text{in}}} \to \sigma(\theta,E) \equiv \frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$$

- Local Normalization

$$\sigma(\theta,E) \underbrace{\to}_{\theta \ll 1} \sigma_{\text{Ruth}}(\theta,E)$$

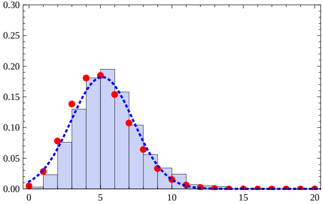
- Total Normalization (mean free path, no Coulomb)

$$l = \frac{1}{n\sigma_T} \quad , \quad \sigma_T = \int d\Omega \frac{d\sigma}{d\Omega}$$

Counting statistics

- Binomial distribution (p scattering probability)

$$P_{N,k} = p^k (1-p)^{N-k} \begin{pmatrix} N \\ k \end{pmatrix} \quad , \quad \langle k \rangle = Np \quad , \quad (\Delta k)^2 = \langle k^2 \rangle - \langle k \rangle^2 = Np(1-p)$$



- Binomial $\rightarrow$ Poisson $\rightarrow$ Gauss

$$P_{N,k} \underbrace{\rightarrow}_{p \ll 1} \frac{e^{-Np}(Np)^k}{k!}$$

$$\underbrace{\rightarrow}_{k \gg 1} \frac{e^{-(k-Np)^2/2}}{\sqrt{2\pi}\Delta k}$$

$$p = 0.1 \quad N = 50$$

-

$$N_{\rm out} = \bar{N}_{\rm out} \pm \Delta N_{\rm out} \quad , \quad \Delta N_{\rm out} = \sqrt{\bar{N}_{\rm out}}$$

- $\sigma(\theta, E)$ is Gauss distributed

$$\sigma(\theta, E) = \bar{\sigma}(\theta, E) \pm \Delta(\theta, E)$$

Partial wave expansion (No spin)

- Scattering amplitude

$$f(\theta,\phi) = \sum_{l=0}^{\infty} (2l+1) \frac{e^{2i\delta_l}-1}{2ip} P_l(\cos\theta) \quad , \qquad E = \frac{p^2}{2\mu}$$

- Schrödinger Equation

$$\left[-\frac{\nabla^2}{2\mu} + V(\vec{x}) \right] \Psi(\vec{x}) = E\Psi(\vec{x})$$

- Spherical symmetry $V(r)$

$$\Psi(\vec{x}) = \frac{u_l(r)}{r} Y_{lm}(\hat{x})$$

- Reduced radial equation

$$-u_l''(r) + \left[\frac{l(l+1)}{r^2} + 2\mu V(r) \right] u_l(r) = p^2 u_l(r)$$

- Asymptotic conditions

$$u_l(r) \underbrace{\to}_{r\to 0} r^{l+1} \quad , \qquad u_l(r) \underbrace{\to}_{r\to\infty} \sin\left(pr - \frac{l\pi}{2} + \delta_l \right)$$

- GOAL : Determine $V(r) \pm \Delta V(r)$

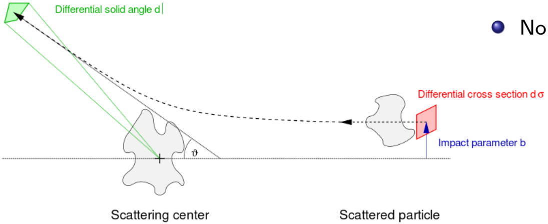
Finite range forces

- Meson exchange picture → Longest range ≡ Lightest particle

$$r_c \sim \frac{\hbar}{m_\pi c} \sim 1.4\text{fm}$$

- Impact parameter

$$|\vec{L}| = |\vec{x} \wedge \vec{p}| \to bp \qquad L^2 = l(l+1)^2 \sim (l+1/2)^2 \to l + \frac{1}{2} = bp$$



- No scattering condition

$$\begin{array}{rcll} V(r) & \sim & 0 & r \gtrsim r_c \\ \delta_l(p) & \sim & 0 & b \gtrsim r_c \\ \to l_{\max} + \frac{1}{2} & \sim & p r_c \sim p/m_\pi & \end{array}$$

- Truncation in the partial wave expansion

$$f(\theta,\phi) = \sum^{l_{\max}} (2l+1) \frac{e^{2i\delta_l} - 1}{2ip} P_l(\cos\theta) = \frac{e^{2i\delta_0} - 1}{2ip} + 3\frac{e^{2i\delta_1} - 1}{2ip} \cos\theta + \dots$$

FITTING

Single energy fits

- Complete data in a GIVEN energy E

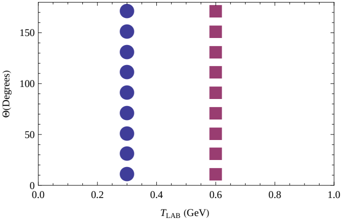
$$\sigma(\theta,E) \to \qquad \sigma(\theta_1,E),\dots,\sigma(\theta_N,E)$$

- Fitting function $\to$ Fitting parameters $\delta_1(E),\dots,\delta_l(E)$

$$\chi^2(\delta_1(E),\dots,\delta_l(E),\nu) = \sum_{i=1}^{N}\left[\frac{\sigma^{\rm exp}(\theta_i,E)-\nu\sigma^{\rm th}(\theta_i,\delta_1(E),\dots,\delta_l(E))}{\Delta\sigma(\theta_i,E)}\right]^2 + \left(\frac{1-\nu}{\Delta\nu}\right)^2$$

- Normalization is COMMON for ONE energy
- Phase-shifts are "experimental" and MODEL INDEPENDENT

$$\delta_l^{\rm exp}(E) \pm \Delta\delta_l^{\rm exp}(E) \quad , \qquad l=0,\dots,l_{\rm max}$$



Multienergy analysis

- Incomplete Data in several energies and angles

$$\sigma(\theta, E) \to \qquad \sigma(\theta_1, E_1), \ldots, \sigma(\theta_N, E_N)$$

- The need for interpolation (Smoothness in (θ, E))
- Fitting function *to* Fitting MODEL DEPENDENT parameters $\mathbf{p} = (p_1, \ldots, p_M)$

$$\chi^2(\mathbf{p}, \nu) = \sum_{i=1}^{N} \left[\frac{\sigma(\theta_i, E_i)^{\rm exp} - \nu \sigma^{\rm th}(\theta_i, E_i, \mathbf{p})}{\Delta\sigma(\theta_i, E_i)} \right]^2 + \left(\frac{1-\nu}{\Delta\nu} \right)^2$$

- The statement

$$\sigma(\theta_i, E_i)^{\rm exp} = \nu_0 \sigma^{\rm th}(\theta_i, E_i, \mathbf{p}_0) \pm \Delta\sigma(\theta_i, E_i)$$

- Too large χ^2/ν
  - Bad model$\to$ SELECT MODEL
  - Bad data $\to$ SELECT DATA
  - Bad model and data

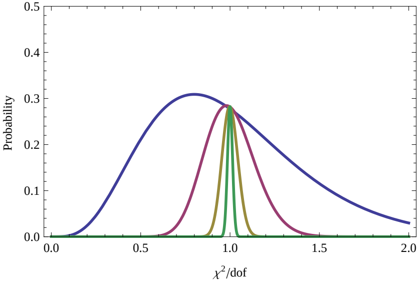
The χ^2-test

- If $\xi_n \in N(0,1)$

$$P_\nu(\chi^2) = \prod_{n=1}^{N} \left(\int_{-\infty}^{\infty} d\xi_i \frac{e^{-\xi_i^2/2}}{\sqrt{2\pi}} \right) \delta(\chi^2 - \sum_{n=1}^{N} \xi_n^2) = \frac{e^{-\chi^2} \chi^{\nu-2}}{2^{\nu/2}\Gamma\left(\frac{\nu}{2}\right)}$$

- Mean and Variance

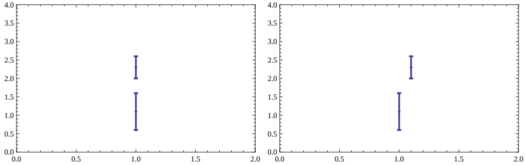
$$\langle \chi^2 \rangle = \nu \,, \qquad \langle (\chi^2 - \langle \chi^2 \rangle)^2 \rangle = 2\nu^2 \,, \qquad \rightarrow \chi^2 = \nu \pm \sqrt{2\nu}$$



ν	$\chi^2/\nu\,(68\%)$
10	1 ± 0.447
100	1 ± 0.141
1000	1 ± 0.044
10000	$1 \pm 0.014.$

The need for selection

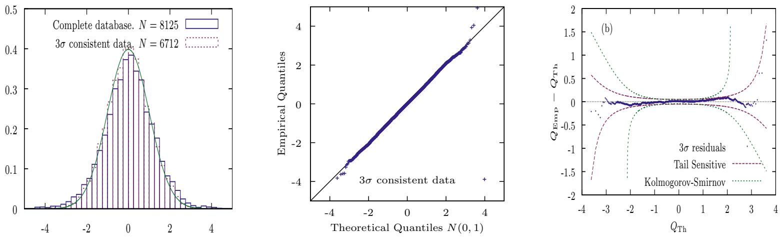
- Example: THE SAID DATABASE
  - PP Data No=25188 $\chi^2 = 48225.043$ (TLAB $\leq$ 3 GeV)
  - NP Data No=12962 $\chi^2 = 26079.973$ (TLAB $\leq$ 3 GeV)
  - πN 41926 Chi2= 166585.05 (TLAB $\leq$ 3 GeV)
  - πN 2599 Chi2= 4586.26 (TLAB $\leq$ 300)
  - πN 1355 Chi2= 2600.75 (10 $\leq$ TLAB $\leq$ 70)
- Which experiments are INCOMPATIBLE ?



- Contribution the χ^2 will be large (discard BOTH ?)
- If errorbar includes BOTH no contribution to the χ^2
- Incompatibility is DESTRUCTIVE
- Real physical effect ?

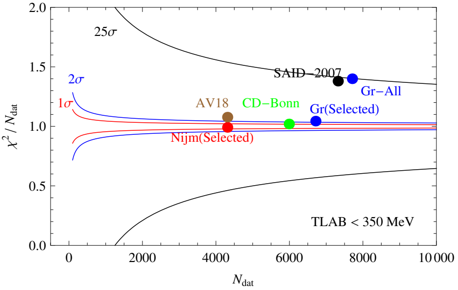
Selection criterium

- Mutually incompatible data. Which experiment is correct? Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  1. Fit to all data $(\chi^2/\nu > 1)$
  2. Remove data sets with improbably high or low χ^2 (3σ criterion)
  3. Refit parameters
  4. Re-apply 3σ criterion to all data
  5. Repeat until no more data is excluded or recovered



To believe or not to believe

$N_{\rm data}$	HJ62	Reid68	TRS75	Paris80	Urb81	Arg84	BonnR	Bonn89	Nijm93
1787	13.5	2.9	3.4	4.5	6.0	7615	1090	25.5	1.8



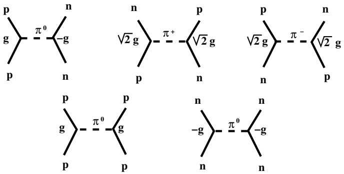
$$\chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu}$$

- Charge dependence in OPE
- Magnetic-Moments, Vacuum polarization, ...

STATISTICAL CONSEQUENCES

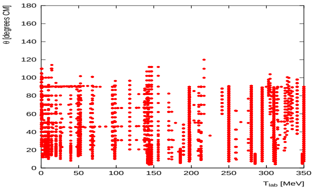
Long distances

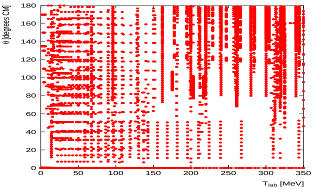
- Nucleons exchange JUST one pion



- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)
- Granada coarse grained analysis (2016) (isospin breaking !!)

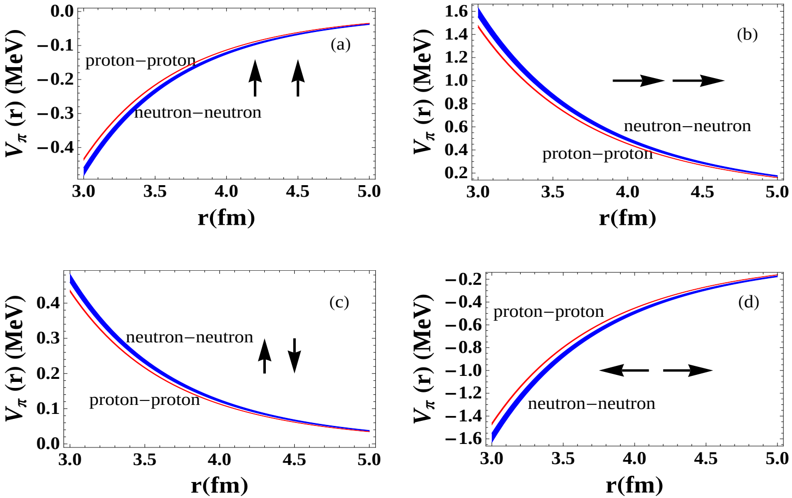
$$g_p^2/(4\pi) = 13.72(7) \neq g_n^2/(4\pi) = 14.91(39) \neq g_c^2/(4\pi) = 13.81(11) \qquad \chi^2/\nu = 1.02$$



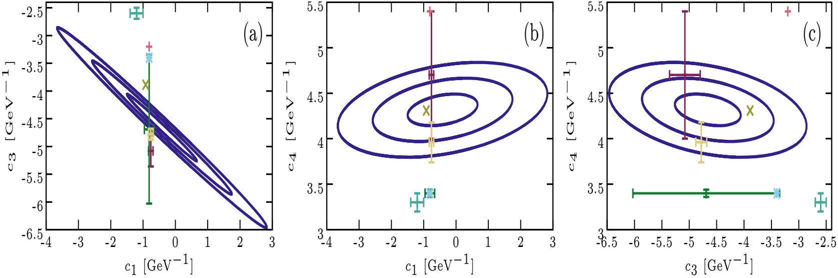


Neutron-Neutron vs Proton-Proton (Polarized)

nn interaction is more intense than pp interaction



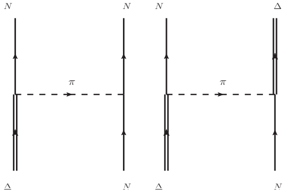
Chiral Two Pion Exchange from Granada-2013 np+pp database



Compatible to Roy-Steiner

$$c_1 = 1.11 \pm 0.03 \quad c_2 = 3.13 \pm 0.03 \qquad c_3 = 5.61 \pm 0.06 \quad c_4 = 4.26 \pm 0.04$$

$f_{\pi N\Delta}$ from Granada-2013 np+pp database



- NN potential in the Born-Oppenheimer approximation

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Fit with $r_e = 1.8\text{fm}$ to $N = 6713 pp + np$ scattering data

$$f_{\pi N\Delta}/f_{\pi NN} = 2.178(14) \qquad \chi^2/\nu = 1.12 \rightarrow h_A = 1.397(9)$$

Frequentist or Bayes ?

- Frequentist: What is the probability that given a theory the data are correct ?
- Bayesian: What is the probability that given the data the theory is correct ? (priors, random parameters)

$$\chi^2_{\rm augmented} \to \chi^2_{\rm data} + \chi^2_{\rm parameters}$$

- Both approaches agree for $N_{\rm Data} \gg N_{\rm Parameters}$. We have $N_{\rm Dat} \sim 7000$ and $N_{\rm Par} = 40$

Maximal energy vs shortest distance

- The full potential is separated into two pieces

$$V(r) = V_{\rm short}(r)\theta(r_c - r) + V^{\chi}_{\rm long}(r)\theta(r - r_c)$$

- Data are fitted up to a maximal $T_{\rm LAB}$

$$T_{\rm LAB} \le \max T_{\rm LAB} \leftrightarrow p_{\rm CM} \le \Lambda$$

Max $T_{\rm LAB}$ MeV	r_c fm	c_1 GeV^{-1}	c_3 GeV^{-1}	c_4 GeV^{-1}	Highest counterterm	χ^2/ν
350	1.8	-0.4(11)	-4.7(6)	4.3(2)	F	1.08
350	1.2	-9.8(2)	0.3(1)	2.84(5)	F	1.26
125	1.8	-0.3(29)	-5.8(16)	4.2(7)	D	1.03
125	1.2	-0.92	-3.89	4.31	P	1.70
125	1.2	-14.9(6)	2.7(2)	3.51(9)	P	1.05

- D-waves, forbidden by Weinberg counting, are indispensable !
- Data and χ-N2LO do not support $r_c < 1.8$fm !
  (Several χ-potentials take $r_c = 0.9 - 1.1$fm)

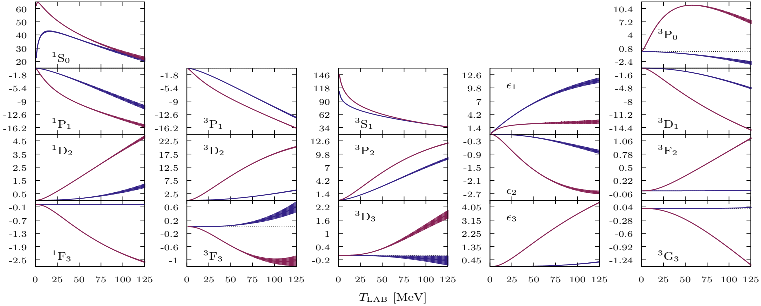
Deconstructing chiral forces

- The full potential is separated into two pieces

$$V(r) = V_{\rm short}(r)\theta(r_c - r) + V^{\chi}_{\rm long}(r)\theta(r - r_c)$$

- Under what conditions are the short distance phases compatible with zero

$$|\delta_{\rm short}| \le \Delta\delta \qquad r_c = 1.8{\rm fm}$$



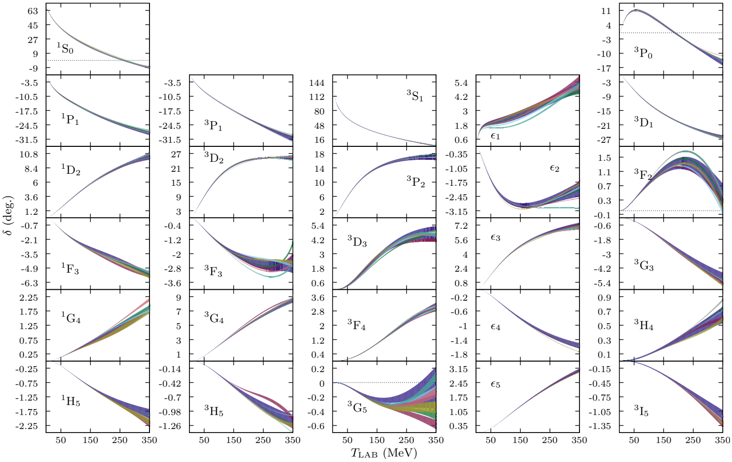
- This is equivalent to set counterterms in given partial waves directly to cero

$$\delta_{\rm short} = 0 \leftrightarrow C_{\rm short} = 0$$

PERIPHERAL TESTS

Statistical vs Systematics

Statistically equivalent interactions with $\chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu}$ DO NOT overlapp



Effective Elementary

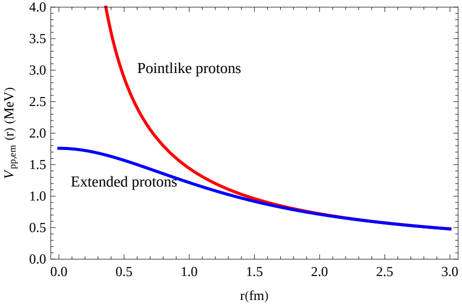
When are two protons interacting as point-like particles ?

- Electromagnetic Form factor

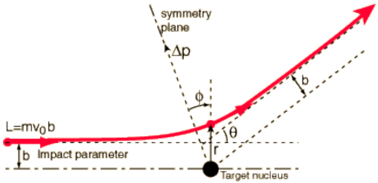
$$F_i(q) = \int d^3 r e^{iq\cdot r} \rho_i(r)$$

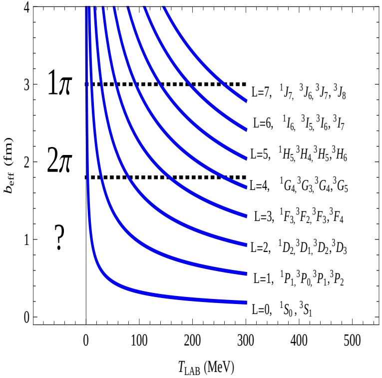
- Electrostatic interaction

$$V_{pp}^{\mathrm{el}}(r) = e^2 \int d^3 r_1 d^3 r_2 \frac{\rho_p(r_1)\rho_p(r_2)}{|\vec{r}_1 - \vec{r}_2 - \vec{r}|} \to \frac{e^2}{r} \qquad r > r_e \sim 2\mathrm{fm}$$



Impact parameter





- Angular momentum conservation

$$L = bp \qquad L^2 = \hbar l(l+1) \approx (l+\frac{1}{2})^2 \qquad p = \hbar k$$

Outliers

- Statistical

$$\xi^i|_{\rm stat} = \frac{\Delta^i - \Delta_{\rm Gr}}{\Delta(\Delta_{\rm Gr})} = \frac{\delta^i - \delta_{\rm Gr}}{\Delta\delta_{\rm Gr}} \, , \tag{1}$$

- Systematic (6 Gr-potentials , 6 Gr+7 HQ (Nijm, CD-Bonn,..)

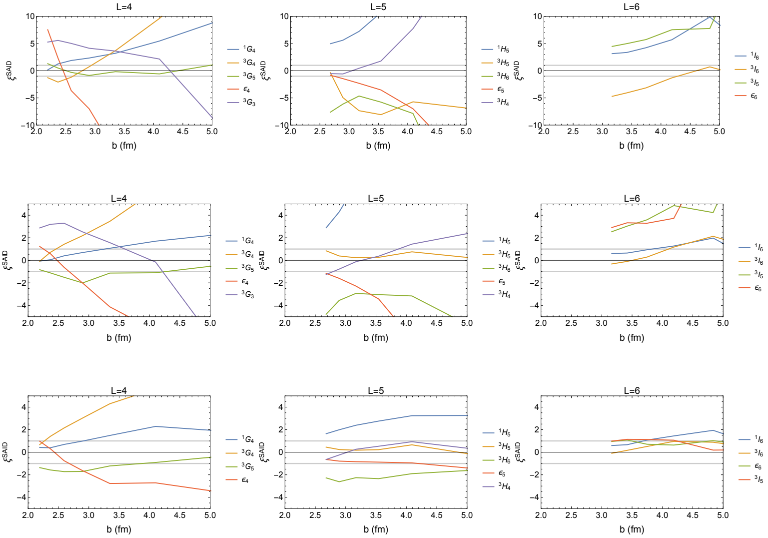
$$\xi^i|_{\rm sys} = \frac{\Delta^i - {\rm Mean}(\Delta)}{{\rm Std}(\Delta)} = \frac{\delta^i - {\rm Mean}(\delta)}{{\rm Std}(\delta)} \, , \tag{2}$$

- Prob. of *not* being an outlier.

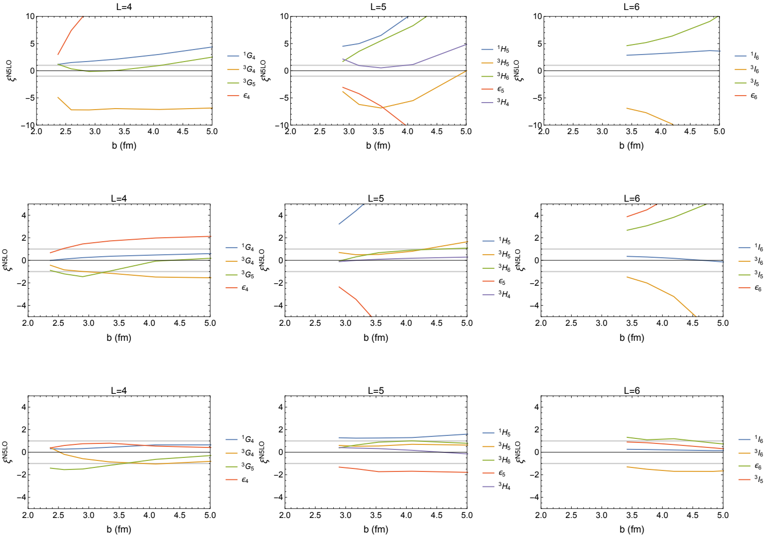
$$p(|\xi| > |\xi_0|) = 1 - \int_{-\xi_0}^{\xi_0} dx \frac{e^{-x^2/2}}{\sqrt{2\pi}} \, . \tag{3}$$

- $\xi_0 = 1, 2, 3$ for $p = 0.32, 0.05, 0.01$

SAID peripheral waves



N5LO peripheral waves



To count or not to count: The Falsification of Chiral Forces

- We can fit CHIRAL forces to ANY energy and look if counterterms are compatible with zero within errors
- We find that if $E_{\rm LAB} \le 125 {\rm MeV}$ Weinberg counting is INCOMPATIBLE with data.
- You have to promote D-wave counterterms.
  N2LO-Chiral TPE + N3LO-Counterterms → Residuals are normal
  Piarulli,Girlanda,Schiavilla,Navarro Pérez,Amaro,RA, PRC
- We find that if $E_{\rm LAB} \le 40 {\rm MeV}$ TPE is INVISIBLE
- We find that peripheral waves predicted by 6th-order chiral perturbation theory ARE NOT consistent with data within uncertainties

$$|\delta^{\rm Ch,N5LO} - \delta^{\rm PWA}| > \Delta\delta^{\rm PWA,stat}$$

5 σ incompatible