Can we reconcile our understanding of the symmetry energy with the isobaric analog state properties?

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Table of contents:

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Theoretical framework for the study of nuclei and nuclear matter:

Nuclear Energy Density Functionals (EDFs) consitute our unique theoretical framework for a microscopic description of the Isobaric Analog State (IAS) energy in medium and heavy nuclei and on the nuclear matter Equation of State (EoS) in a consistent way.

$$\langle \Psi | \mathcal{H} | \Psi \rangle \approx \langle \Phi | \mathcal{H}_{\text{eff}} | \Phi \rangle = E[\rho] \ \textbf{(?)}$$

• Commonly derived from an effective Hamiltonian solved at the Hartree-Fock level.

Nuclear Energy Density Functionals:

Main types of successful EDFs are derived from Hartree-Fock (mean-field) calculations based on an effective interaciton

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

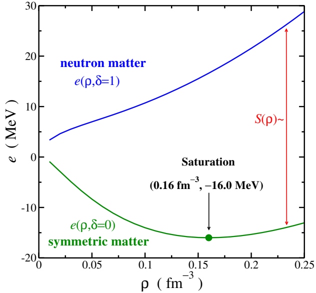
$$
\begin{aligned}
\mathcal{L}_{\mathtt{int}} \quad & = \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi},\Psi)\Psi\Phi_{\sigma} && +\bar{\Psi}\Gamma_{\delta}(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_{\delta} \\
& -\bar{\Psi}\Gamma_{\omega}(\bar{\Psi},\Psi)\gamma_{\mu}\Psi A^{(\omega)\mu} && -\bar{\Psi}\Gamma_{\rho}(\bar{\Psi},\Psi)\gamma_{\mu}\tau\Psi A^{(\rho)\mu} \\
& -e\bar{\Psi}\hat{Q}\gamma_{\mu}\Psi A^{(\gamma)\mu}
\end{aligned}
$$

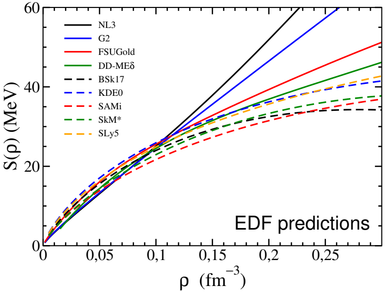
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\rm eff}_{\rm Nucl} = V^{\rm long-range}_{\rm attractive} + V^{\rm short-range}_{\rm repulsive} + V_{\rm SO}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System





* The nuclear EoS can be written in good approximation as:

$$e(\rho,\beta) \approx e(\rho,\beta=0) + S(\rho)\beta^2 \text{ where } \beta \equiv \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

* SNM can be expanded around ρ_0 and define some useful parameters:

$$e(\rho,0) \approx e(\rho_0,0) + K\varepsilon^2 \text{ where } \varepsilon \equiv \frac{\rho_0 - \rho}{3\rho_0}$$

* Symmetry energy can be also expanded around ρ_0 and define some useful parameters:

$$S(\rho) \approx J - L\varepsilon + K_{sym}\varepsilon^2$$

Examples: EoS parameters from nuclear observables

EDFs provide a good description of nuclear masses ($\sim 0.1\%$), charge radii ($\sim 0.1 - 1\%$) and collective oscillation frequencies around the g.s. (GR excitation energies)

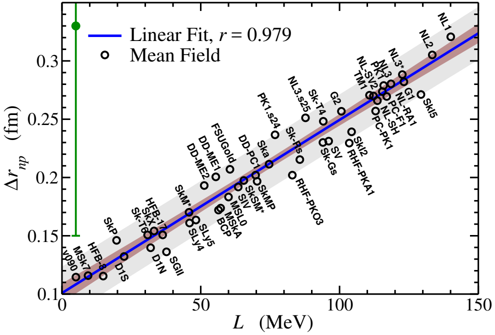
Some physical insights may be obtained from simple considerations (models) while studyting microscopically a given observable:

- $B(A, Z)$ determine very precisely $e(\rho_0, 0) \equiv e_0$ (-16 MeV)
$$\frac{\delta e_0}{e_0} \sim 1\% \Rightarrow \delta B(^{208}\mathrm{Pb}) \sim 30 \text{ MeV } (2\%)$$
A small change on $e(\rho_0, 0)$ *will predict unrealistic* B *in a heavy nucleus*
- The **interior density (ρ_0)** in most of existing nuclei is 0.16 fm^{-3}
$$\frac{\delta \rho_0}{\rho_0} \sim 5\% \Rightarrow \delta r(^{208}\mathrm{Pb}) \sim \delta r_0 A^{1/3} \sim 0.1 \text{ fm } (2\%)$$
A small change will predict not very good radii and this will/may also affect B

Examples: EoS parameters from nuclear observables

Isovector properties (e.g. $S(\rho)$) are thought to be well determined by the **neutron skin thickness** ($\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$) **of a heavy nucleus such as ^{208}Pb)**:

Macroscopic model: $\Delta r_{np} \sim \dfrac{1}{12} \dfrac{(N-Z)}{A} \dfrac{R}{J} L \qquad (L \propto p_0^{neut})$



Physical Review Letters **106**, *252501 (2011)*

[Exp. from strongly interacting probes: $\sim 0.15 - 0.22$ fm (*Physical Review C* 86 *015803 (2012)*)].

Micorscopic models (EDFs) confirm such a relation

However the experimental precision and accuracy needed in the measurment of this property is very challenging nowadays.

Isovector properties in nuclei

- **In the past**, **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes** and $(N-Z)/A$ **explored is small**.

$\Downarrow$

Limited knowledge of isovector properties

(Not precise: model dependent analysis due to incomplete understanding of the strong interaction in the low-energy regime important for nuclei.)

- **At present**,
  - the use of **RIBs** has opened the possibility of measuring properties of **nuclei with large** $N-Z$
  - **parity violating elastic electron scattering**, a **model independent technique**, has allowed to estimate the **weak (neutron) form factor at low** q of ^{208}Pb

$\Downarrow$

Promising perspectives for the near future

1) We need to reliably assess the quality of our extrapolations

2) Find observables that are not sensitive to the strong force

Covariance analysis: χ^2 test

- Observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$
$$\chi^2(\mathbf{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\text{theo.}} - \mathcal{O}_\imath^{\text{ref.}}}{\Delta \mathcal{O}_\imath^{\text{ref.}}} \right)^2$$
- Assuming that the χ^2 can be approximated by an hyper-parabola around the minimum $\mathbf{p}_0$,
$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{\imath,\jmath}^{n} (p_\imath - p_{0\imath}) \partial_{p_\imath} \partial_{p_\jmath} \chi^2 (p_\jmath - p_{0\jmath})$$
where $\mathcal{M} \equiv \frac{1}{2} \partial_{p_\imath} \partial_{p_\jmath} \chi^2$ (curvature m.) and $\mathcal{E} \equiv \mathcal{M}^{-1}$ (error m.).
- errors between predicted observables $\mathcal{A}$
$$\Delta \mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A}$$
- correlations between predicted observables,
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}}$$
where, $C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$

Example: 2 different fitting protocols and models:

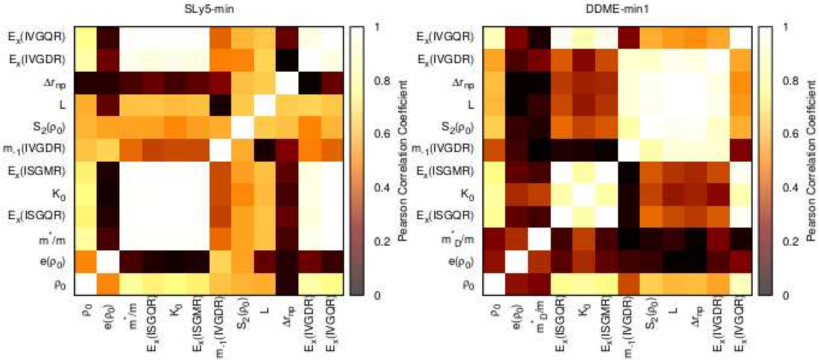
SLy5-min: use constant error for a given observable

- **Binding energies** of ^{40,48}Ca, ^{56}Ni, ^{130,132}Sn and ^{208}Pb with a fixed adopted error of **2 MeV**
- the **charge radius** of ^{40,48}Ca, ^{56}Ni and ^{208}Pb with a fixed adopted error of **0.02 fm**
- the **neutron matter** Equation of State calculated by Wiringa *et al.* (1988) for densities between 0.07 and 0.40 fm^{-3} with an adopted error of **10%**
- the **saturation energy** ($e(\rho_0) = -16.0 \pm 0.2$ MeV) and **density** ($\rho_0 = 0.160 \pm 0.005$ fm^{-3}) of symmetric nuclear matter.

DD-ME-min1: use relative error for all observables

- **binding energies, charge radii, diffraction radii and surface thicknesses** of 17 even-even spherical nuclei, ^{16}O, ^{40,48}Ca, ^{56,58}Ni, ^{88}Sr, ^{90}Zr, ^{100,112,120,124,132}Sn, ^{136}Xe, ^{144}Sm and ^{202,208,214}Pb. The assumed errors of these observables are **0.2%, 0.5%, 0.5%, and 1.5%**, respectively.

Covariance analysis: SLy5-min and DD-ME-min1



• L and the neutron skin thickness in ^{208}Pb are shown to be correlated.

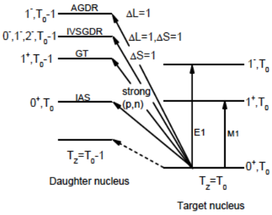
• L and the energy of the Isobaric Analog State are also correlated (not shown here)

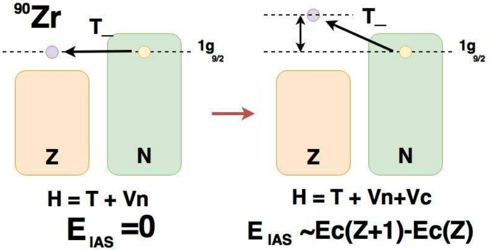
Covariance analysis: SLy5-min and DD-ME-min1

	SLy5-min			DDME-min1			
A	A_0		$\sigma(A_0)$	A_0		$\sigma(A_0)$	units
SNM							
ρ_0	0.162	$\pm$	0.002	0.150	$\pm$	0.001	fm^{-3}
$e(\rho_0)$	-16.02	$\pm$	0.06	-16.18	$\pm$	0.03	MeV
m^*/m	0.698	$\pm$	0.070	0.573	$\pm$	0.008	
J	32.60	$\pm$	0.71	33.0	$\pm$	1.7	MeV
K_0	230.5	$\pm$	9.0	261	$\pm$	23	MeV
L	47.5	$\pm$	4.5	55	$\pm$	16	MeV
^{208}Pb							
Δr_{np}	0.1655	$\pm$	0.0069	0.20	$\pm$	0.03	fm

- **Statistical error on Δr_{np} (and IAS energy) much smaller than systematic error**
- In addition, **statistical uncertainties** depend on the fitting protocol, that is on the **data (or pseudo-data) and associated errors used for the fits**

The isobaric analog state energy: ΔE_d





- **Definition:** $(N, Z+1) \to (N+1, Z)$: T_0 g.s. isospin of $(N+1, Z)$, its IAS in $(N, Z+1)$ will be the lowest state where $T = T_0$.
- **Analog state** can be defined: $|A\rangle = \dfrac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy**

$$E_{IAS} \approx \Delta E_d \equiv E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|[T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

E_{IAS}^{exp} easy to measure and depends only on isospin symmetry symmetry breaking terms: Coulomb and to less extent (few %) strong interaction

The displacement energy: contributions

$[\mathcal{H}, T_-] \neq 0$? essentially **Coulomb potential** but not only

Table: Estimate of the different effects on ΔE_d in ^{208}Pb. Physical Review Letters **23**, 484 (1969).

	ΔE_d Correction
Coumb direct	$\sim$ 20 MeV
Coulomb exchange	$\sim$ -300 keV
n-p mass difference	$\sim$ tens keV
Electromagnetic spin-orbit	$\sim$ - tens keV
Finite size effects	$\sim$ - 100 keV
Short range correlations	$\sim$ 100 keV
Isospin impurity	$\sim$ -100 keV
Isospin symmetry breaking	$\sim$ - 250 keV
	$\sim$ 19 MeV

$E^{exp}_{IAS} = 18.826 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

Coulomb direct displacement energy

$$\Delta E_d \approx \Delta E_d^{C,\mathrm{direct}} = \frac{1}{N-Z}\int \left[\rho_n(\vec{r}) - \rho_p(\vec{r})\right] U_C^{\mathrm{direct}}(\vec{r})\, d\vec{r}$$

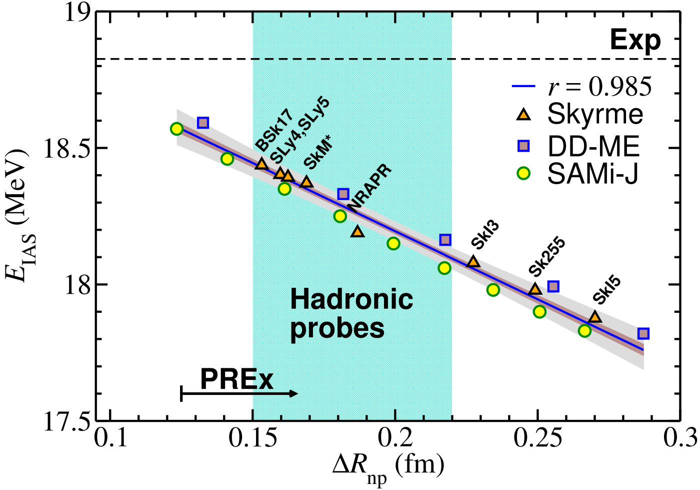
where $U_C^{\mathrm{direct}}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)\, d\vec{r}_1$

Assuming a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$\Delta E_d \approx \Delta E_d^{C,\mathrm{direct}} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \frac{1}{2}\frac{N}{N-Z}\frac{R_n - R_p}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

E_{IAS} in Energy Density Functionals (No Corr.)



EDFs derived from Hartree-(Fock) + Random Phase approximations using relativistic (and non-relativistic) interactions where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

Corrections: For the first time within self-consistent HF+RPA

Within the **HF+RPA** one can **estimate** the E_{IAS} accounting (in an effective way) for **short-range correlations, isospin impurities and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\text{exact}}\varphi_i(\vec{r}) = -\frac{e^2}{2}\int d^3r' \, \frac{\varphi_j^*(\vec{r}')\varphi_j(\vec{r})}{|\vec{r}-\vec{r}'|}\varphi_i(\vec{r}')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\text{emso}} = \frac{\hbar^2 c^2}{2m_i^2 c^4}\langle \vec{l}_i \cdot \vec{s}_i\rangle x_i \int \frac{1}{r}\frac{dU_C}{dr}|R_i(r)|^2$$

where x_i: $g_p - 1$ for Z and g_n for N; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \rightarrow R_{nl}$ radial wf.

Corrections:

- **Finite size** effects (assuming spherical symmetry):

$$
\begin{aligned}
\rho_{ch}(q) &= \left(1 - \frac{q^2}{8m^2}\right)\left[G_{E,p}(q^2)\rho_p(q) + G_{E,n}(q^2)\rho_n(q)\right] \\
&- \frac{\pi q^2}{2m^2}\sum_{l,t}\left[2G_{M,t}(q^2) - G_{E,t}(q^2)\right]\langle \vec{l}\cdot\vec{s}\rangle \int_0^\infty dx \frac{j_1(qx)}{qx}|R_{nl}(x)x^2|^2
\end{aligned}
$$

- The lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$ consists on the selfenergy and the **vacuum polarization** corrections:

$$
V_{vp}(\vec{r}) = -\frac{2}{3}\frac{\alpha e^2}{\pi}\int d\vec{r}'\,\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathcal{K}_1\left(\frac{2}{\lambda\!\!\!^{-}_e}|\vec{r}-\vec{r}'|\right)
$$

where e is the fundamental electric charge, α the fine-structure constrant, $\lambda\!\!\!^{-}_e$ the reduced Compton electron wavelength and

$$
\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt}\left(\frac{1}{t^2} + \frac{1}{2t^4}\right)\sqrt{t^2-1}
$$

Corrections:

- **Isospin symmetry breaking** (Skyrme-like): **two parts**

charge symmetry breaking $+$ **charge independence breaking***

$$V_{CSB} = V_{nn} - V_{pp}$$

$$V_{CIB} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$$

$$V_{CSB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}[\tau_z(1) + \tau_z(2)]\Big\{s_0(1 + y_0 P_\sigma)$$
$$+\frac{1}{2}s_1(1 + y_1 P_\sigma)\left[P'^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)P^2\right]$$
$$+s_2(1 + y_2 P_\sigma)\vec{P}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{P}\Big\}$$

where $\vec{P} \equiv \frac{1}{2\imath}(\vec{\nabla}_1 - \vec{\nabla}_2)$ acts on the right and P' is its complex conjugate acting on the left and $P_{\tau/\sigma}$ are the usual projector operators in isospin and spin spaces.

$$V_{CIB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2)\Big\{u_0(1 + z_0 P_\sigma)$$
$$+\frac{1}{2}u_1(1 + z_1 P_\sigma)\left[P'^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)P^2\right]$$
$$+u_2(1 + z_2 P_\sigma)\vec{P}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{P}\Big\}$$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1) \cdot \vec{\tau}(2)$. Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be fitted!**

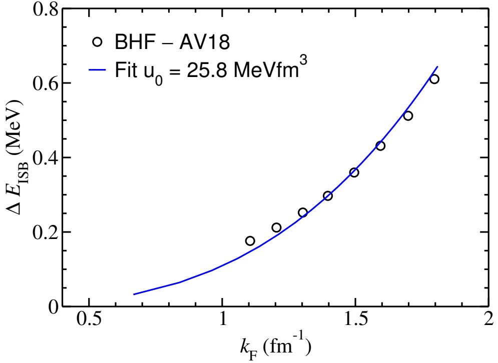
Isospin symmetry breaking in the medium:

- **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**



Physics Letters B 445, 259 (1999)

Re-fit of SAMi: SAMi-ISB

- All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.** $\Rightarrow$ a **re-fit of the interaction is needed**.
- Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159(1)	0.1613(6)	fm^{-3}
e_∞	−15.93(9)	−16.03(2)	MeV
m^*_{IS}	0.6752(3)	0.730(19)	
m^*_{IV}	0.664(13)	0.667(120)	
J	28(1)	30.8(4)	MeV
L	44(7)	50(4)	MeV
K_∞	245(1)	235(4)	MeV

SAMi-ISB finite nuclei properties

El.	N	B [MeV]	B^{exp} [MeV]	r_c [fm]	r_c^{exp} [fm]	ΔR_{np} [fm]
Ca	28	417.67	415.99	3.49	3.47	0.214
Zr	50	783.60	783.89	4.26	4.27	0.097
Sn	82	1102.75	1102.85	4.73	–	0.217
Pb	126	1635.78	1636.43	5.50	5.50	0.151

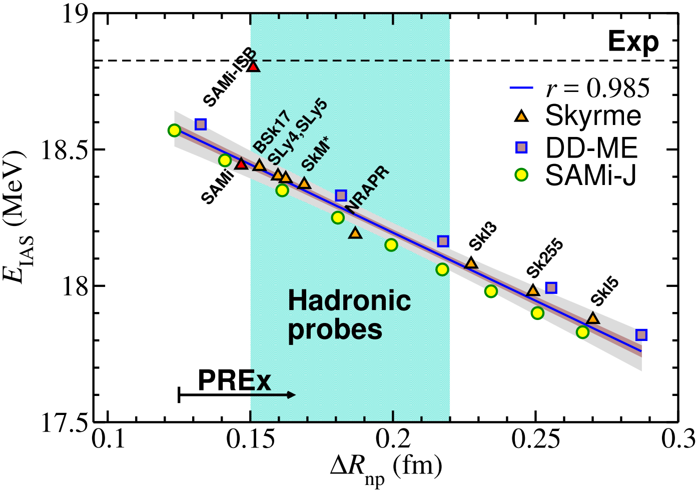
Corrections on E_{IAS} for ^{208}Pb one by one

	E_{IAS} [MeV]	Correction [keV]
No corrections[a]	18.31	
Exact Coulomb exchange	18.41	+100
n/p mass difference	18.44	+30
Electromagnetic spin-orbit	18.45	+10
Finite size effects	18.40	-50
Vacuum polarization (V_{ch})	18.53	+130
Isospin symmetry breaking	**18.80**	+270

[a] From Skyrme Hamiltonian where the nuclear part is isospin symmetric and V_{ch} is calculated from the ρ_p

$E_{IAS}^{exp} = 18.826 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

E_{IAS} with SAMi-ISB



Conclusions: EoS around saturation

- → The **isovector channel** of the nuclear effective interaction is **not well constrained by current experimental information.**
- → Many **observables available in current laboratories** are sensitive to the symmetry energy. **Problems**: **accuracy and model dependent analysis (precision)**. **Systematic experiments** may help on the accuracy.
- → **Exotic nuclei more sensitive** to the isovector properties (due to larger neutron excess). **Problems**: more **difficult to measure, accuracy and model dependent analysis (precision)**. **Systematic experiments** may help accuracy.
- → The most promising observables to constraint the symmetry energy are those that can be measured via processes with little or no influence of the strong force ⇒ **precision**: Δr_{np}, α_D, E_{IAS} (if ISB in the medium were better understood) and the ΔR_{ch} **between mirror nuclei** (**intimately connected with the IAS**) in **medium and heavy nuclei**.

Conclusions: Isobaric Analog State

- **EDFs** of common use in nuclear physics show a **linear dependence between E_{IAS} and Δr_{np}**
- **EDFs** do **not properly** describe the experimental E_{IAS}
- **Modification of $\mathcal{H}_{eff}$ requires a refit** of the interaction including **new ISB parameters**.
- **One can reconcile good reproduction of experimenatal charge radii, binding energies, E_{IAS}...**

- **A better knowledge of ISB contributions in the medium may lead to an accurate determination of neutron skin thickness via E_{IAS} (or the other way around)**

Thank you for your attention!

E_{IAS} in Energy Density Functionals