^{7}Be(p,γ)^{8}B: how EFT and Bayesian analysis can improve a reaction calculation

Daniel Phillips

Work done in collaboration with: K. Nollett (SDSU), X. Zhang (UW)

Phys. Rev. C 89, 051602 (2014), Phys. Lett. B751, 535 (2015), EPJ Web Conf. 113 , 06001 (2016), arXiv:1708.04017

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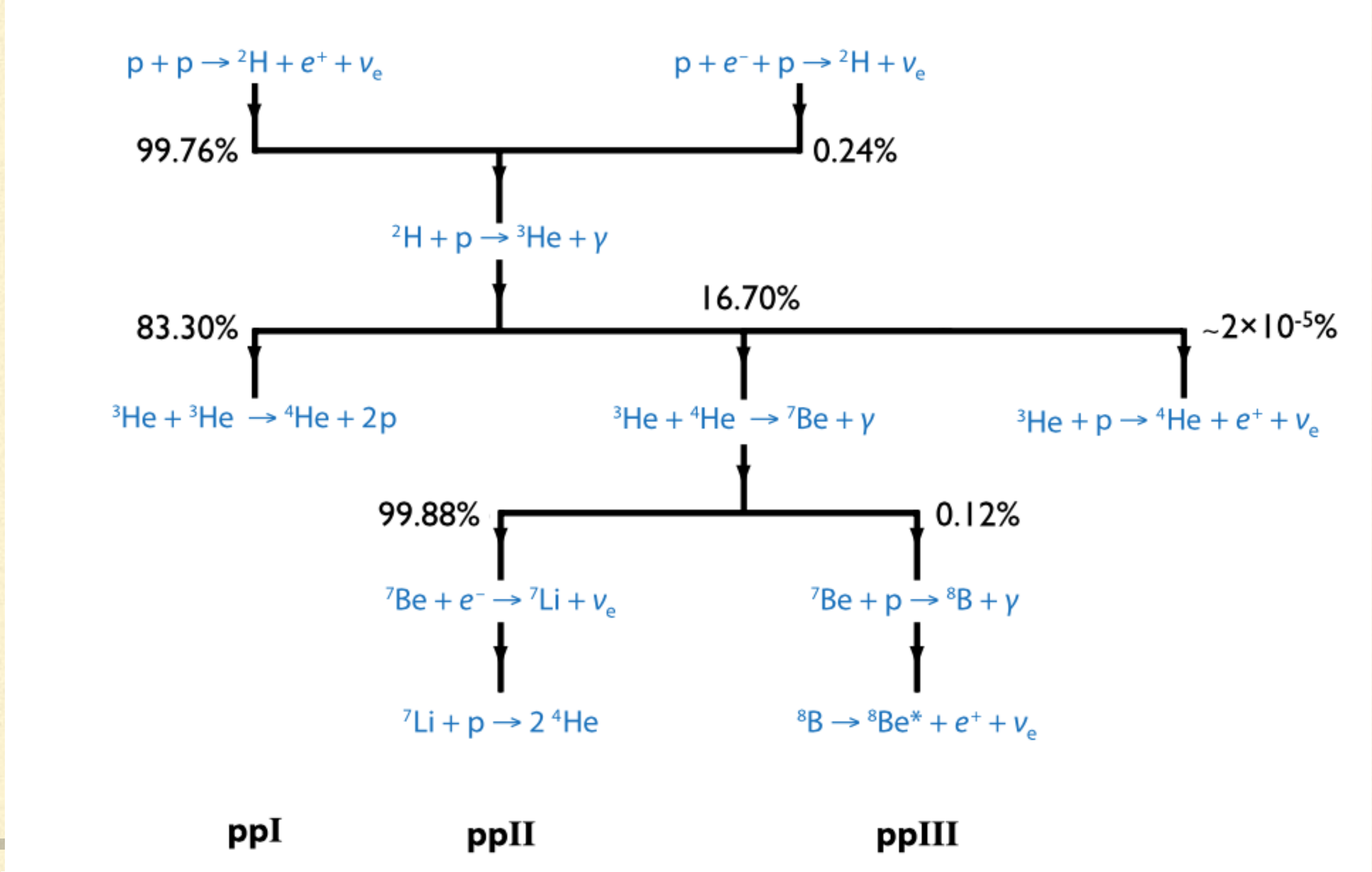
Research supported by the US Department of Energy

Why is ^{7}Be + p→^{8}B + γ important?

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)



$p + p \rightarrow {}^{2}H + e^{+} + \nu_e$

$p + e^{-} + p \rightarrow {}^{2}H + \nu_e$

99.76%

0.24%

${}^{2}H + p \rightarrow {}^{3}He + \gamma$

16.70%

83.30%

~2×10-5%

${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$

${}^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

${}^{3}He + p \rightarrow {}^{4}He + e^{+} + \nu_e$

99.88%

0.12%

${}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$

${}^{7}Be + p \rightarrow {}^{8}B + \gamma$

${}^{7}Li + p \rightarrow 2\,{}^{4}He$

${}^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu_e$

ppI

ppII

ppIII

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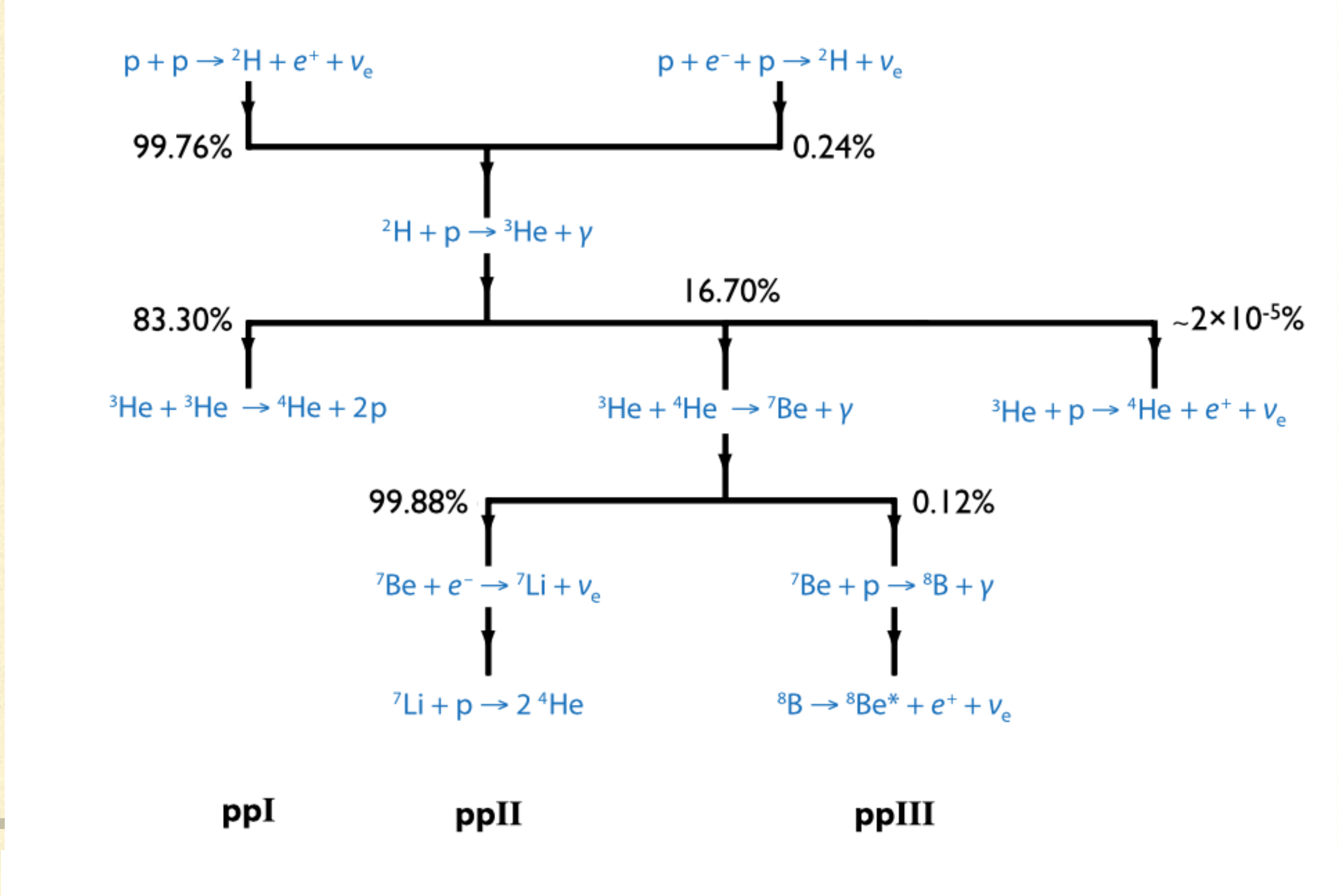
- Part of pp chain (ppIII)



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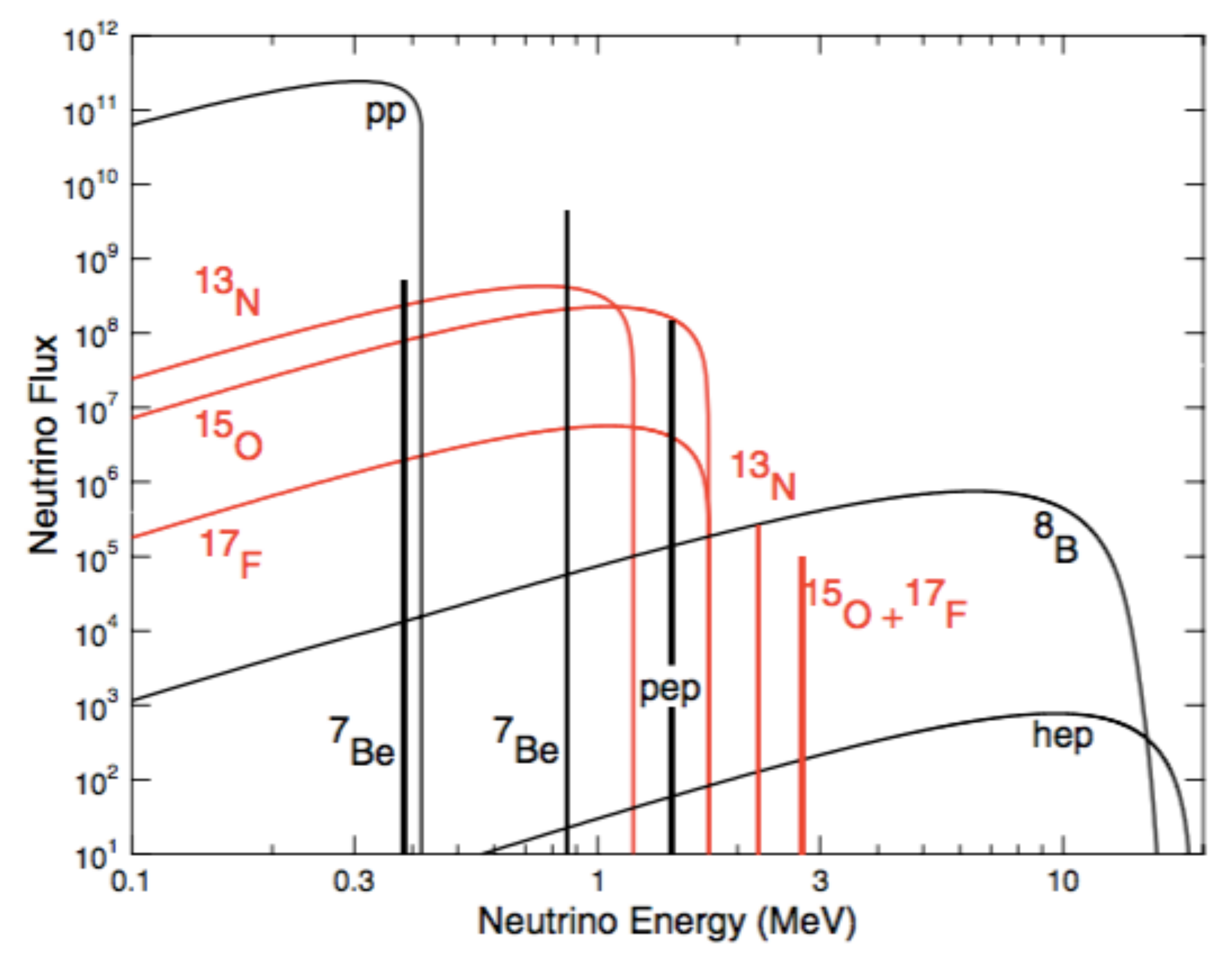

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- Part of pp chain (ppIII)
- Key for predicting flux of solar neutrinos, especially high-energy (^{8}B) neutrinos
- Accurate knowledge of ^{7}Be(p,γ) needed for inferences from solar-neutrino flux regarding solar composition →solar-system formation history

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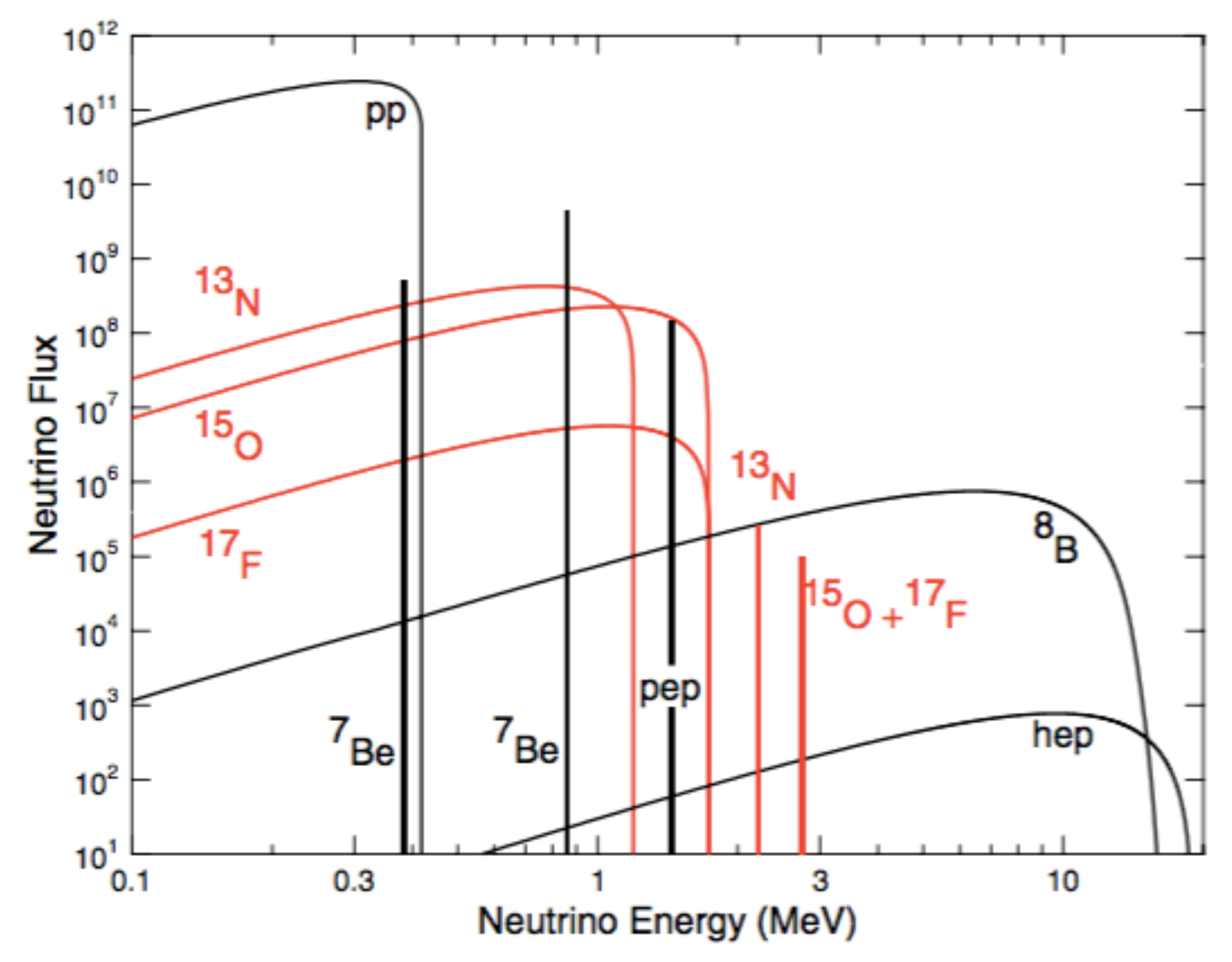

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Thermonuclear reaction rate $\propto \langle v\sigma \rangle \propto \int_0^\infty dE \, \exp\left(-\frac{E}{k_B T}\right) E \, \sigma(E)$

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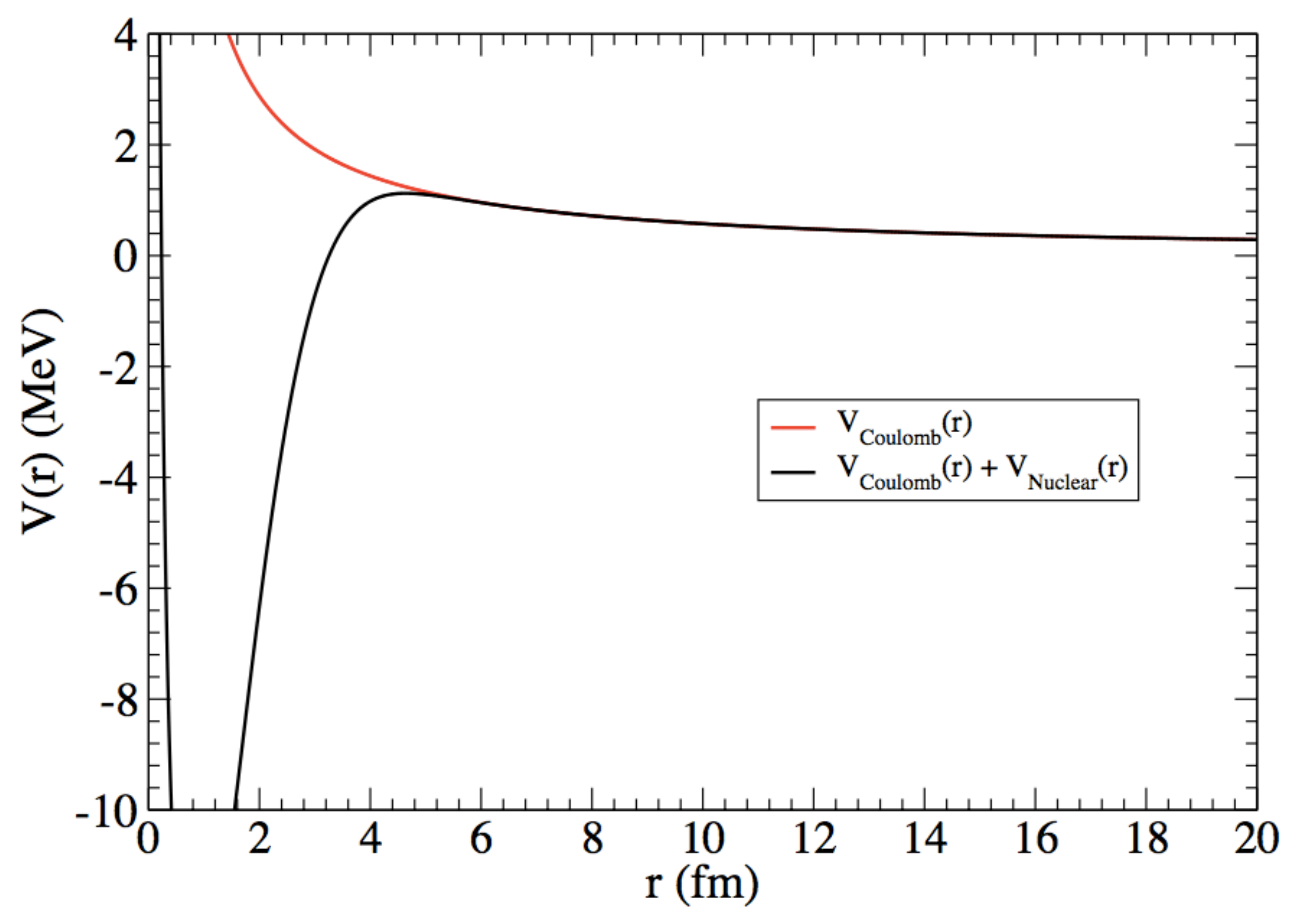

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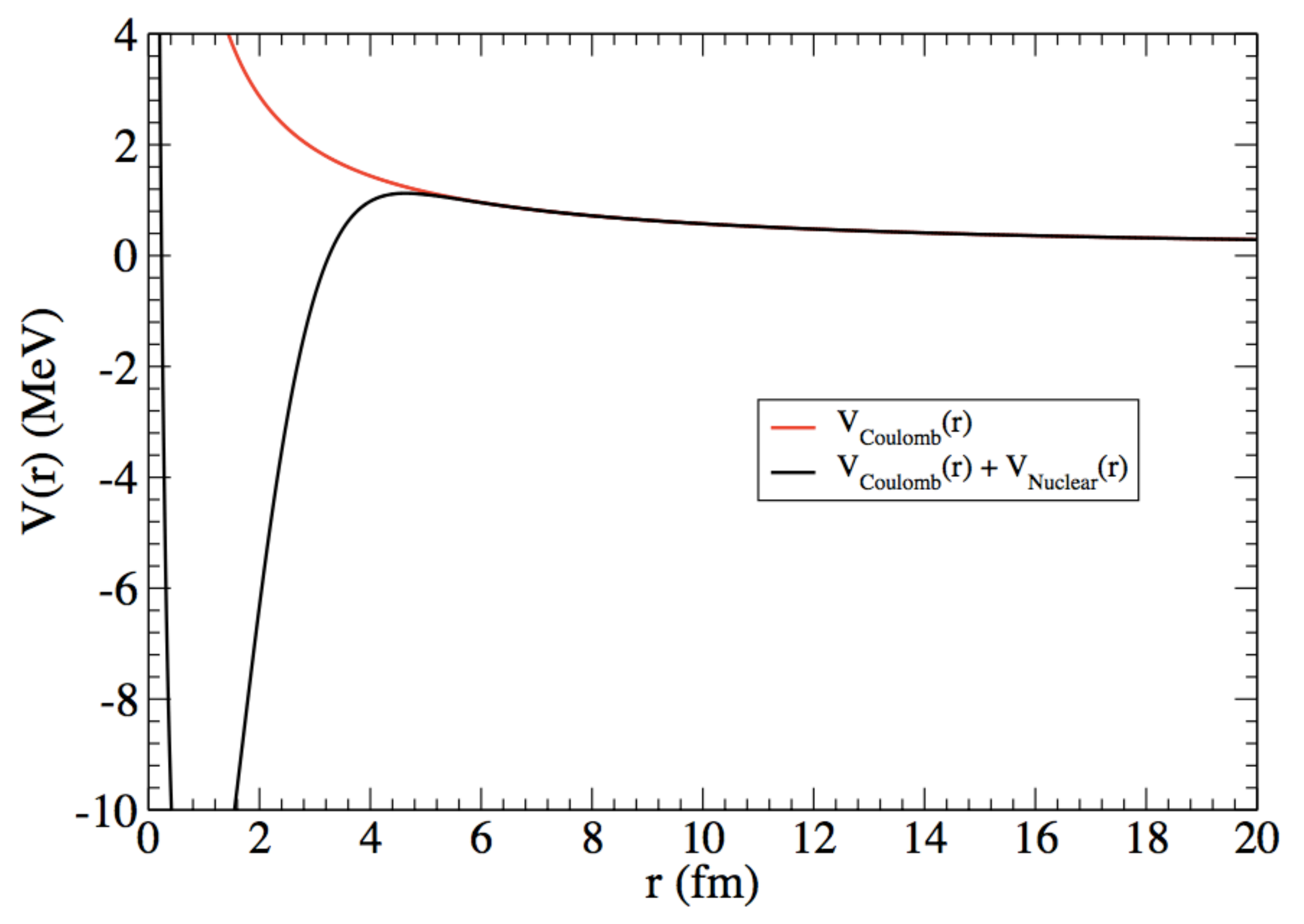

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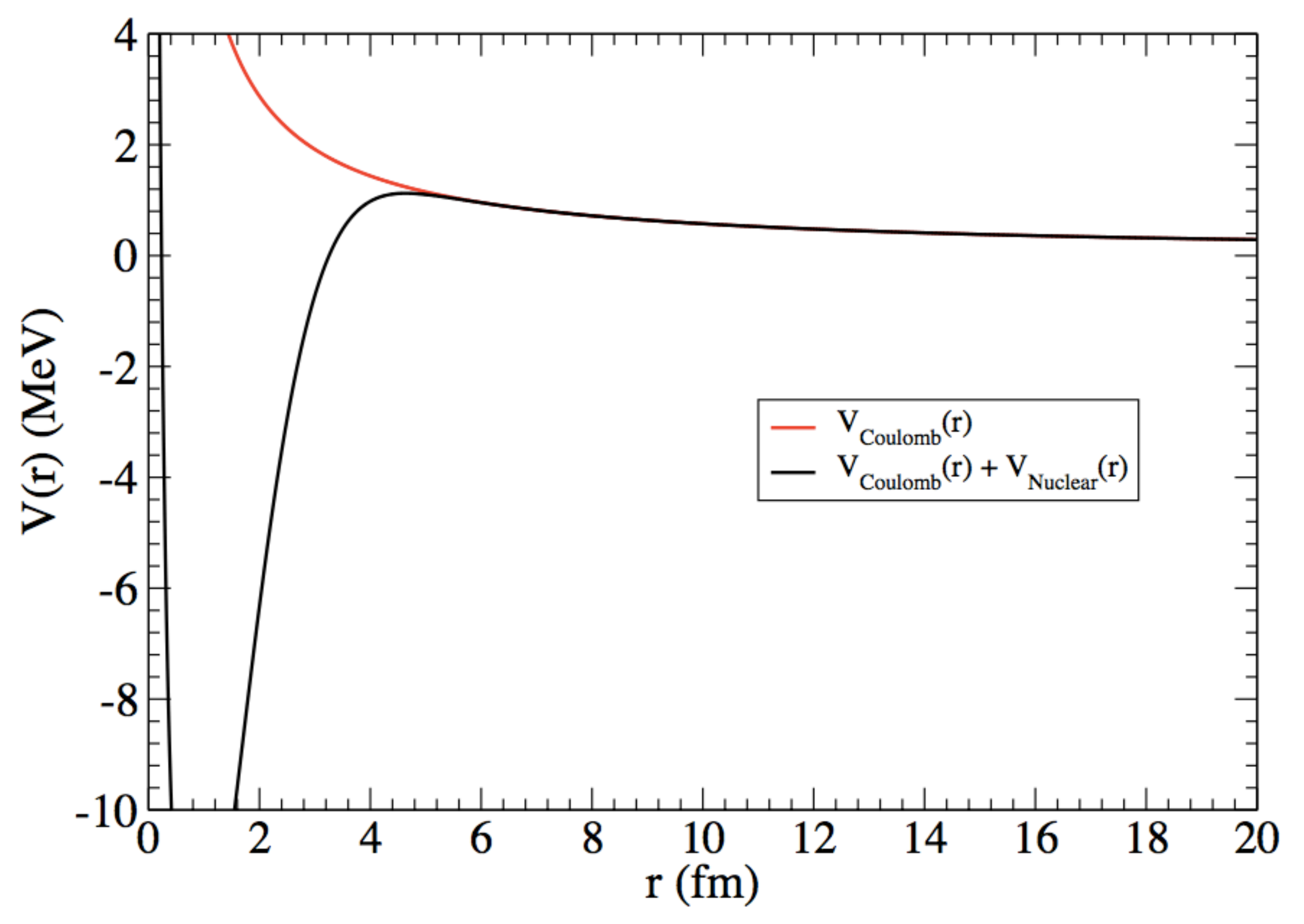

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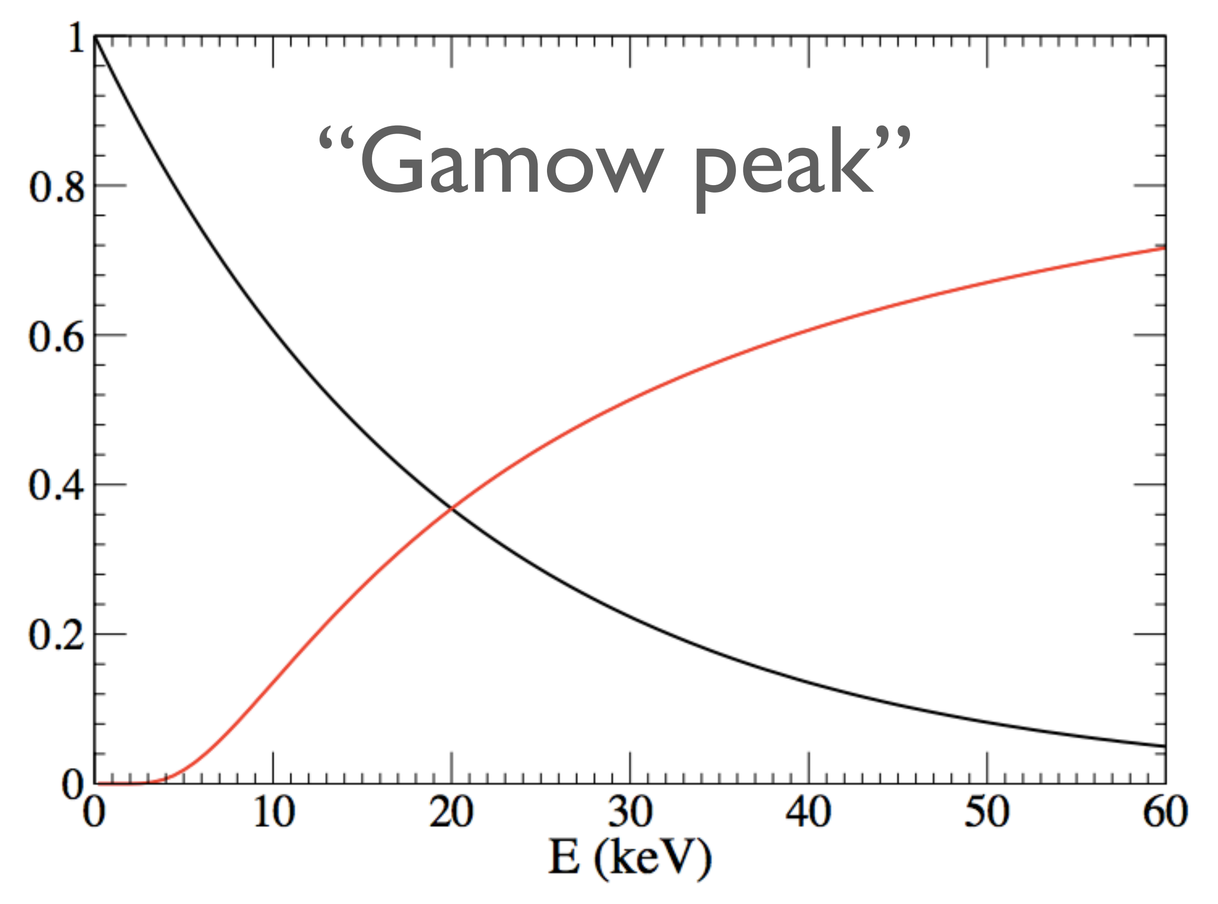

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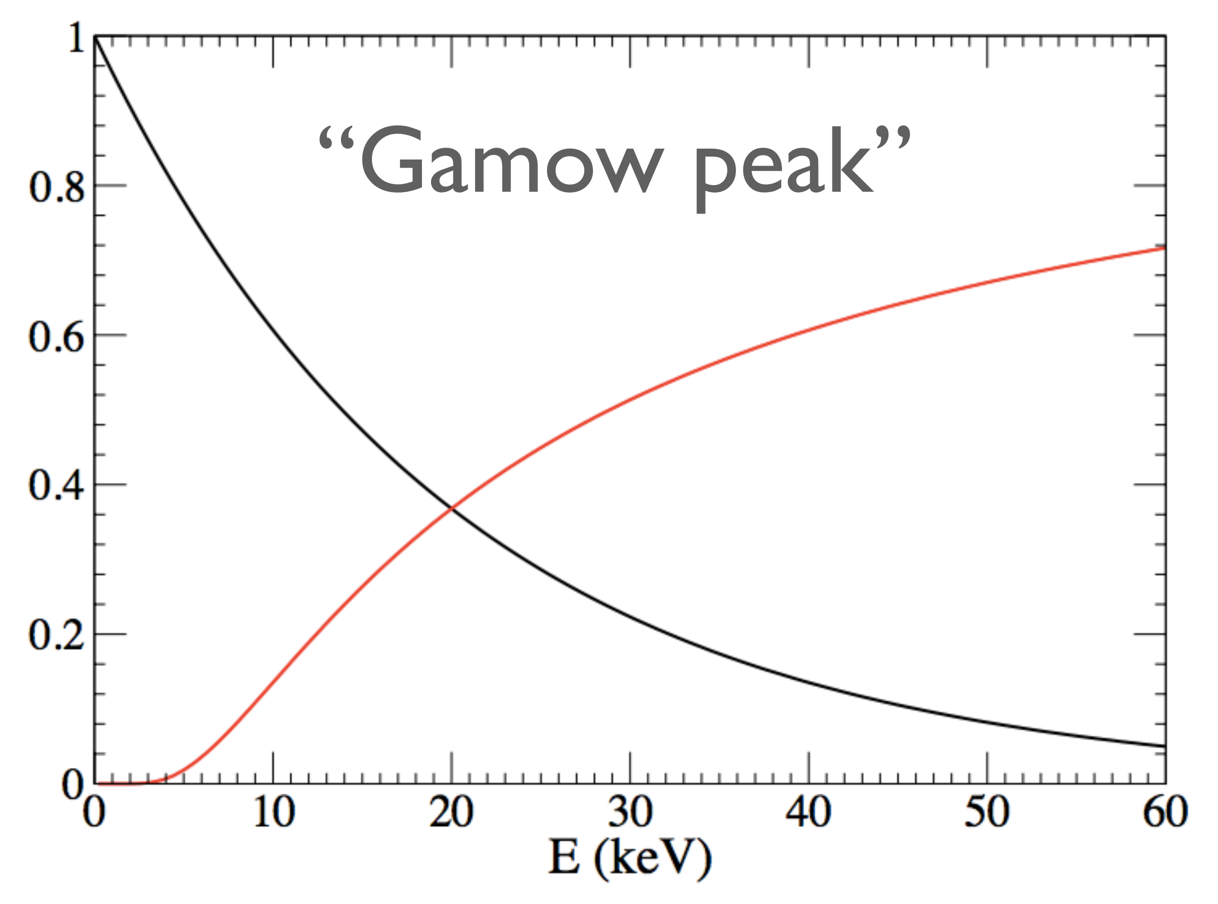
- E1 capture: ^{7}Be + p→^{8}B + γ
- Energies of relevance 20 keV



Outline

- ^{7}Be + p→^{8}B + γ is an important extrapolation problem
- What parameters govern the extrapolation? What is the standard extrapolation method?
- A more reliable extrapolant from Halo Effective Field Theory
- NLO Halo EFT + Bayesian analysis→a better extrapolation
- Summary and future work

What matters where?

$$\mathcal{M}(E) \propto \int dr A_1 \exp(-\gamma_1 r) \left(1 + \frac{1}{\gamma_1 r}\right) r u_E(r); \quad \gamma_1 = 1/(13 \text{ fm})$$

Dominated by ^{7}Be-p separations ~10s of fm

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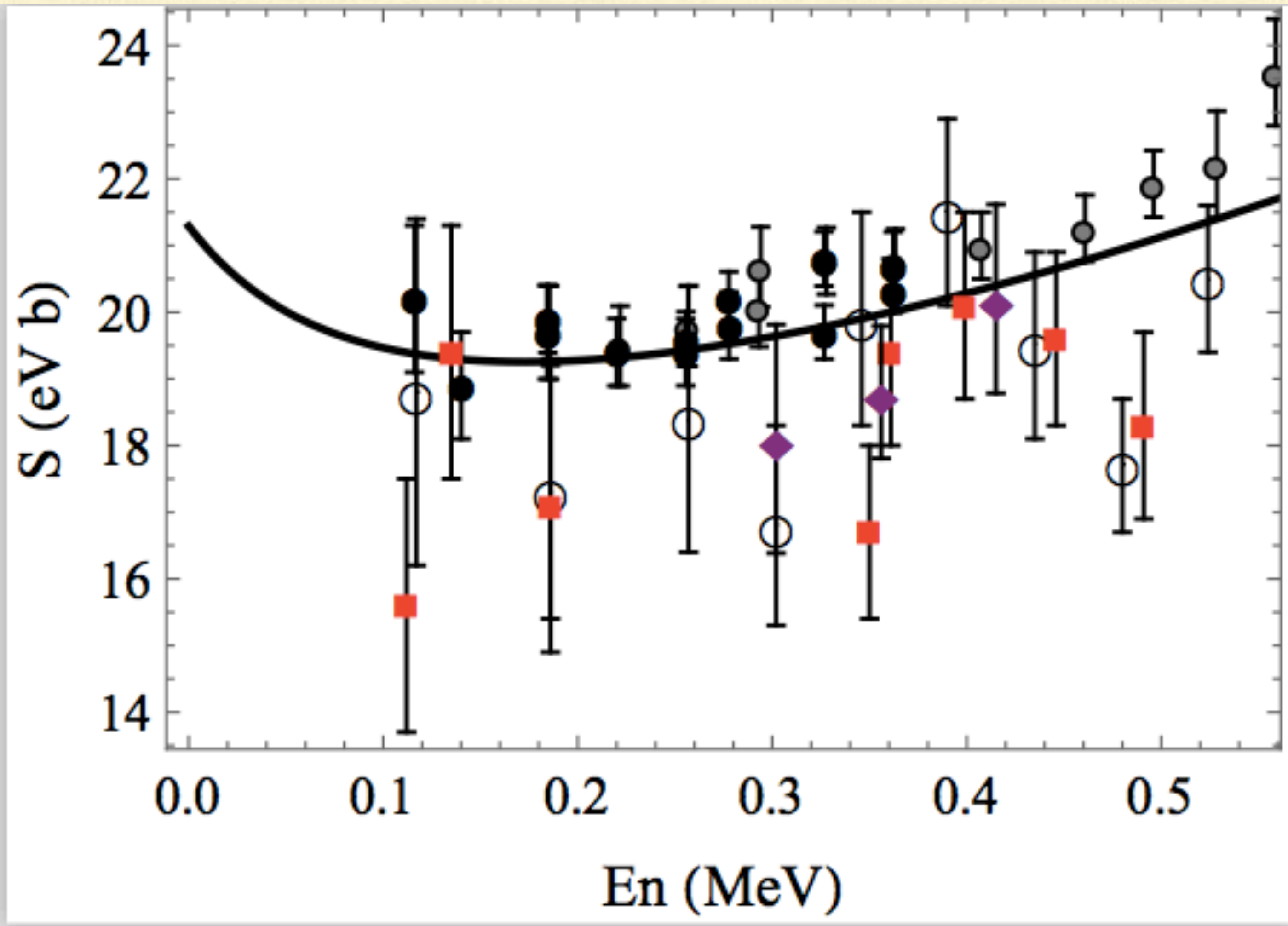

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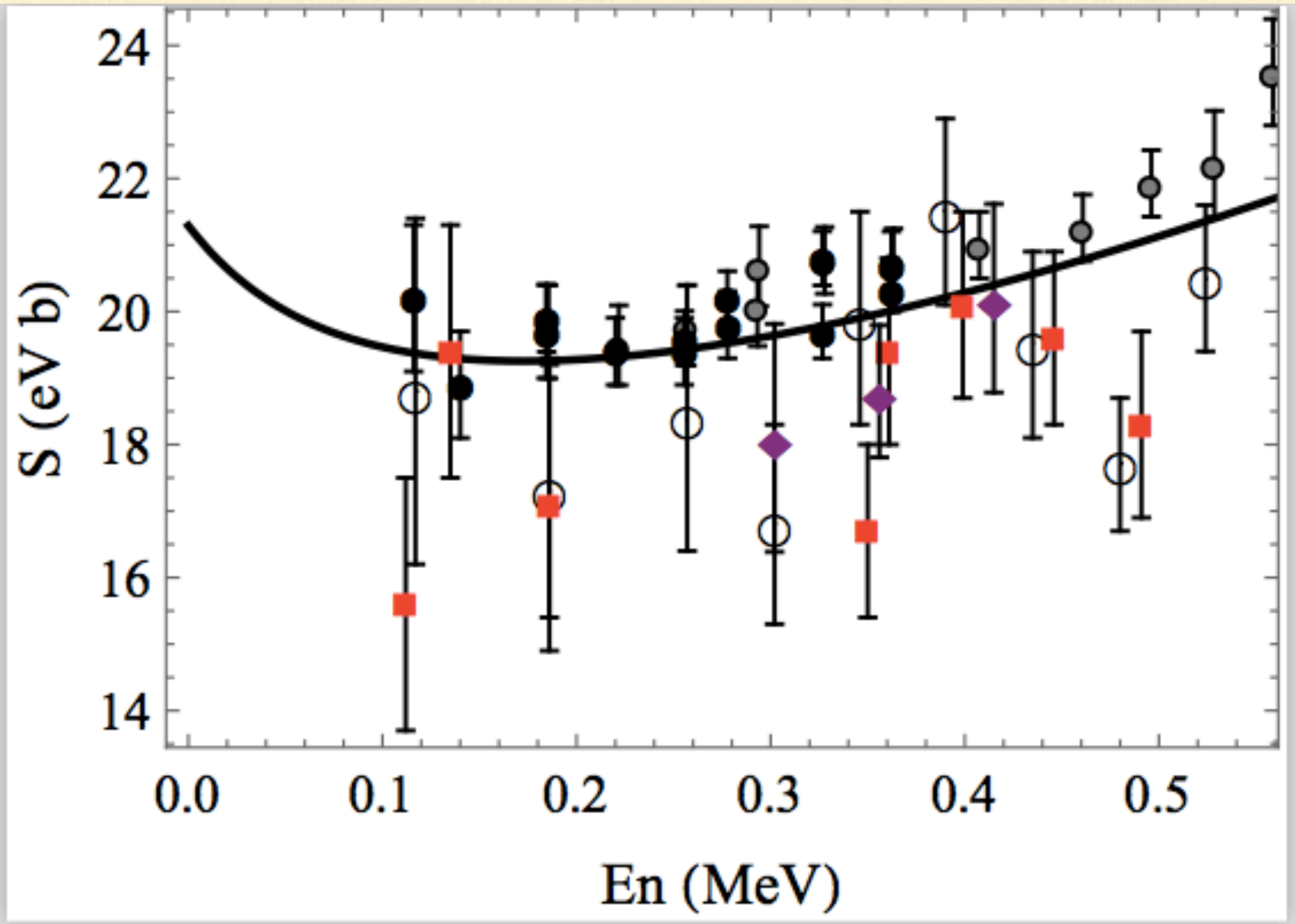

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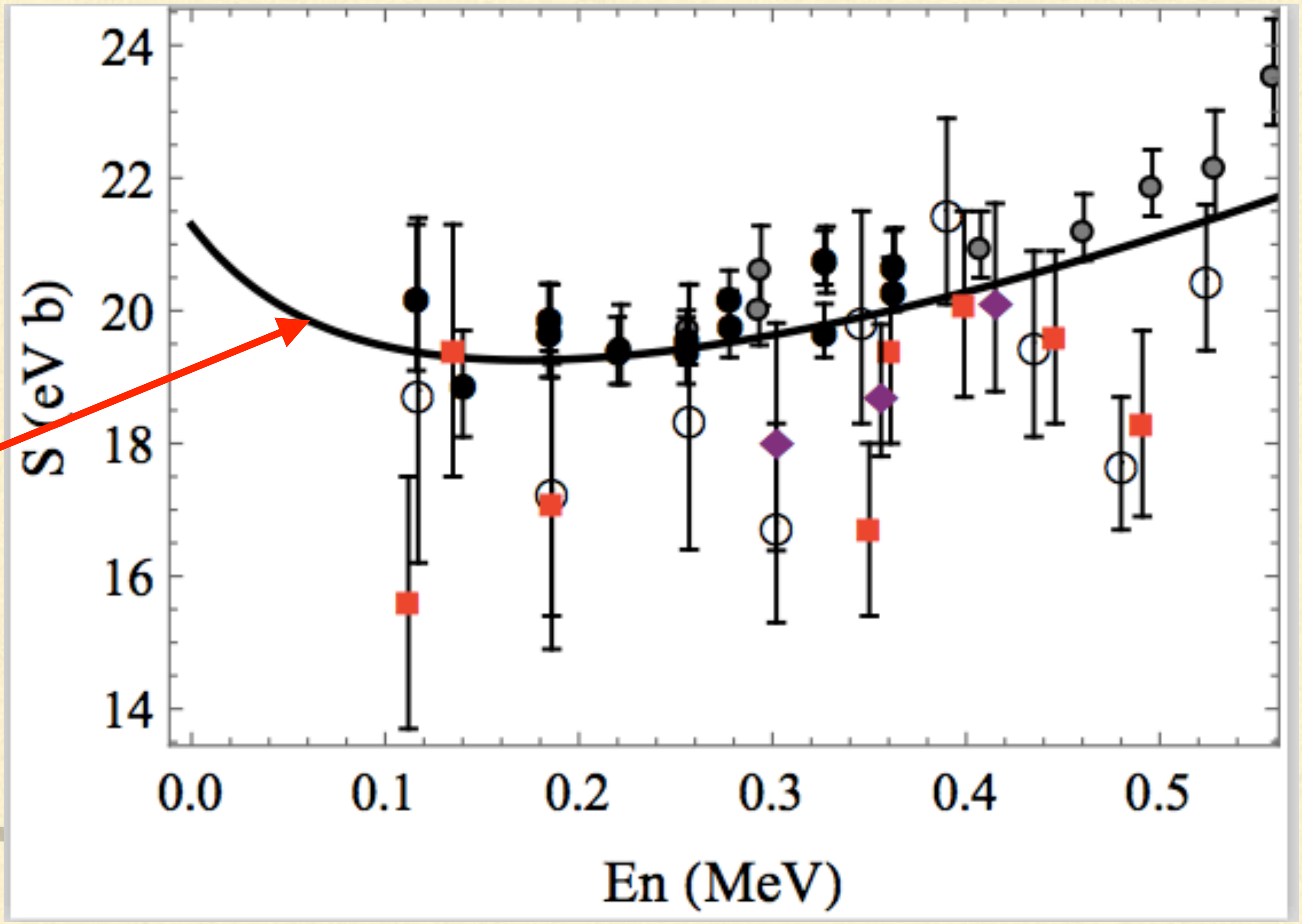

Bound state (ANC &γ_1)
+ Coulomb

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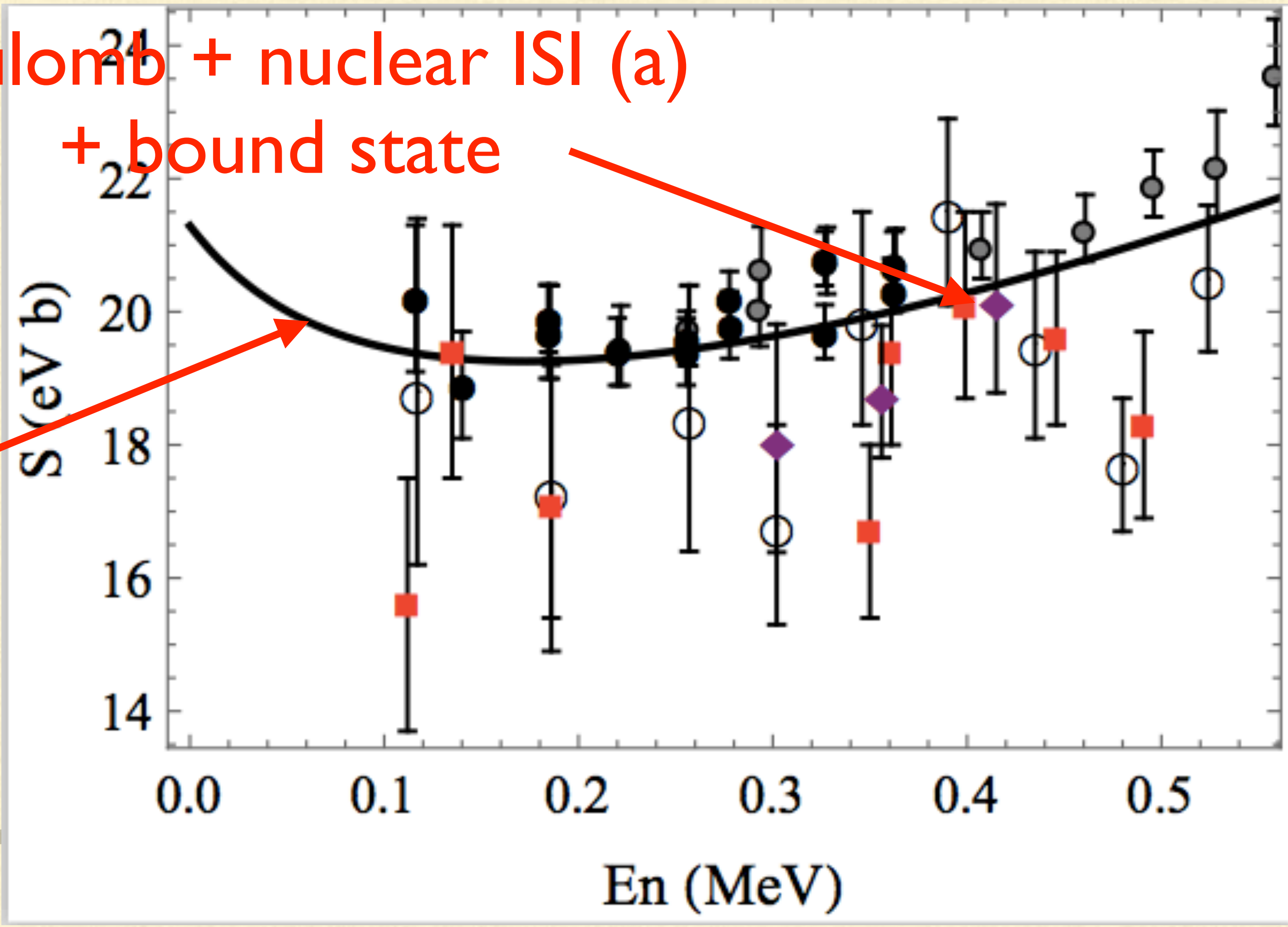

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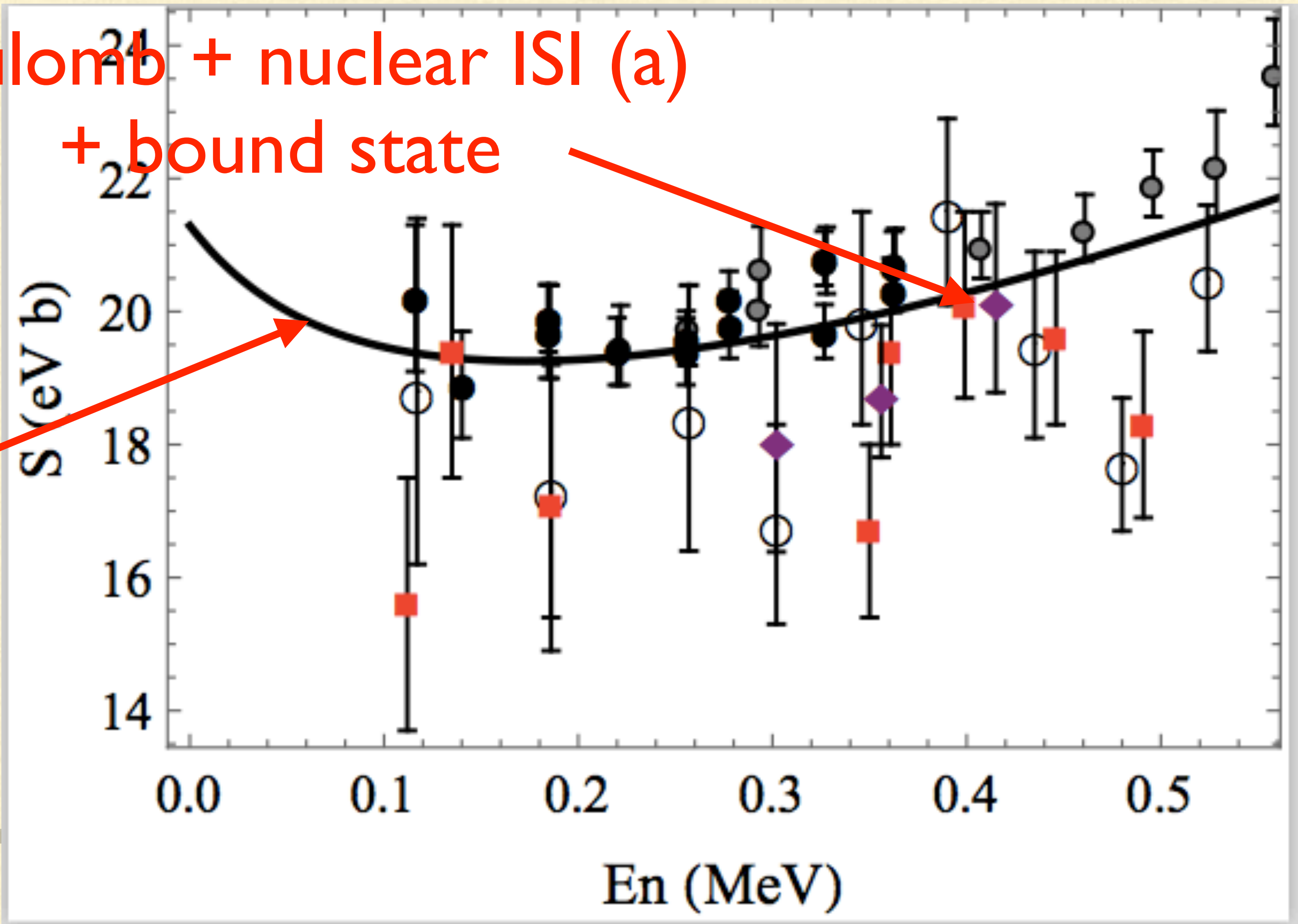
- Sub-leading polynomial behavior in E/E_{core}



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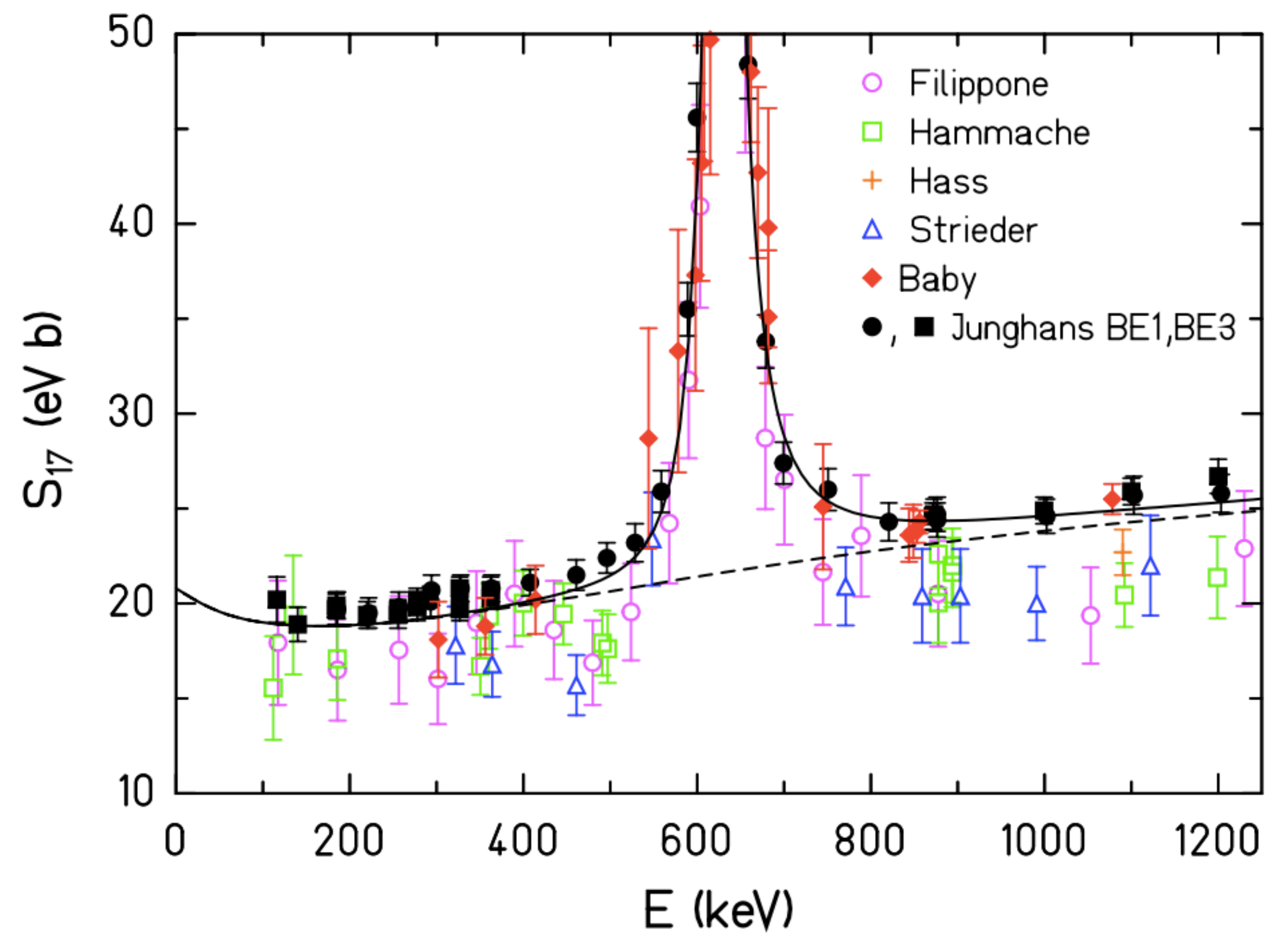
Status as of 2012

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Below narrow 1^+ resonance proceeds via s- and d-wave direct E1 capture
- Energy dependence due to interplay of Coulomb and strong forces
- "Solar fusion II": community evaluation of cross sections relevant for pp and CNO cycles



- SF II value: S(0)=20.8±0.7±1.4 eV b SF I: $S(0)=19^{+4}_{-2}$ eV b
- Used energy dependence from a "best" calculation. Errors from consideration of energy-dependence in a variety of "reasonable models"

Effective Field Theory

- Simpler theory that reproduces results of full theory at long distances
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g., multipole expansion
- Expansion in ratio of physical scales: $p/\Lambda_b=\lambda_b/r$
- Symmetries of underlying theory limit possibilities: all possible terms up to a given order present in EFT
- Short distances: unknown coefficients at a given order in the expansion need to be determined. Symmetry relates their impact on different processes
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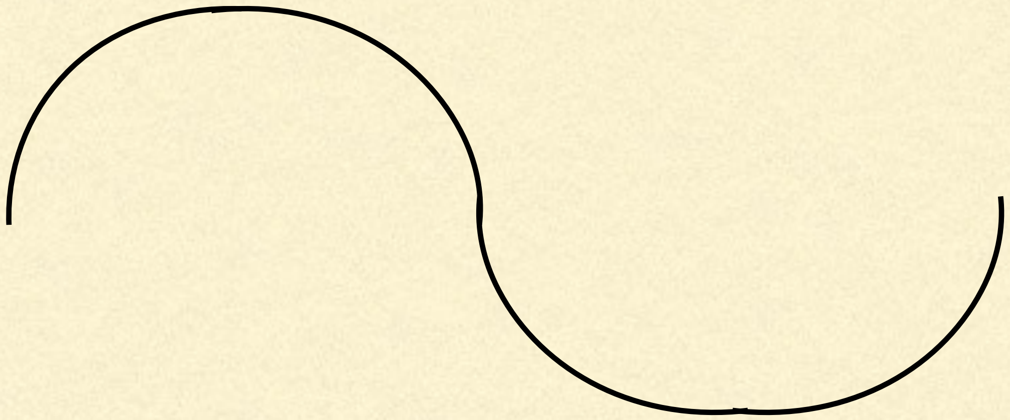
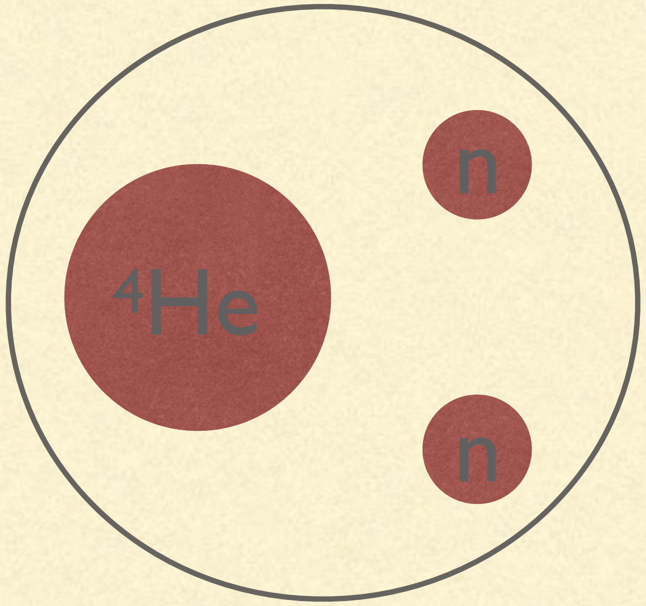
Error grows as first omitted term in expansion

Halo EFT

$\lambda \gg R_{core};\ \lambda \lesssim R_{halo}$





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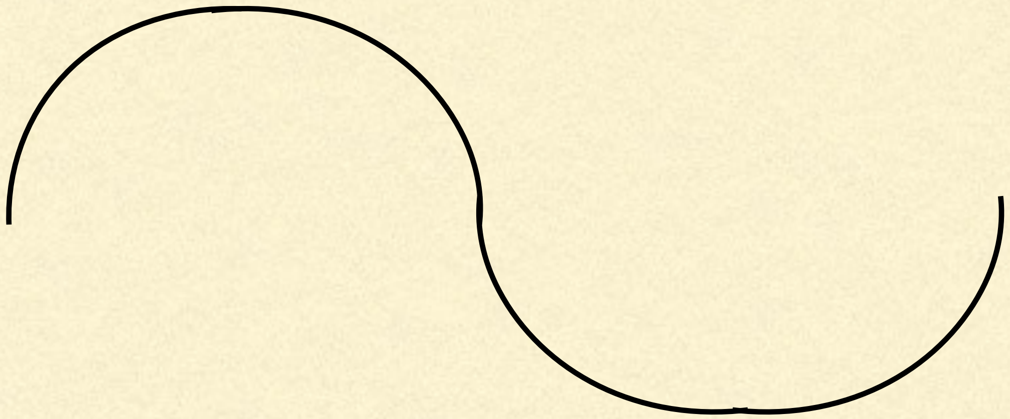
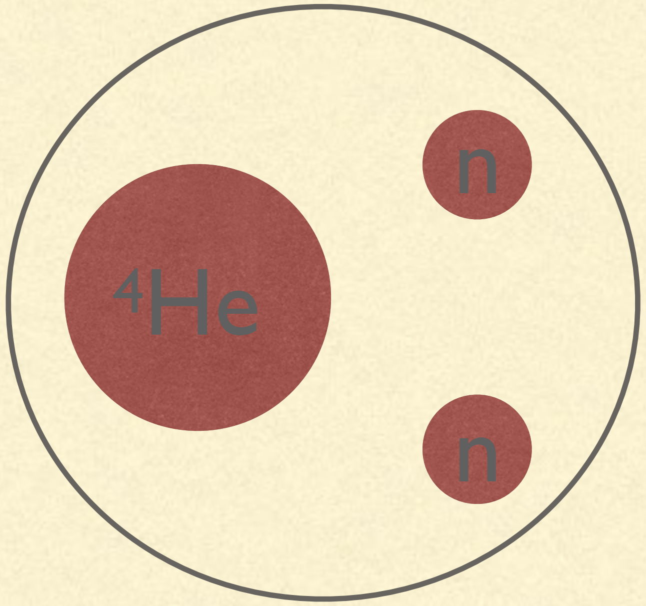




- Define $R_{halo}=\langle r^2\rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2\rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

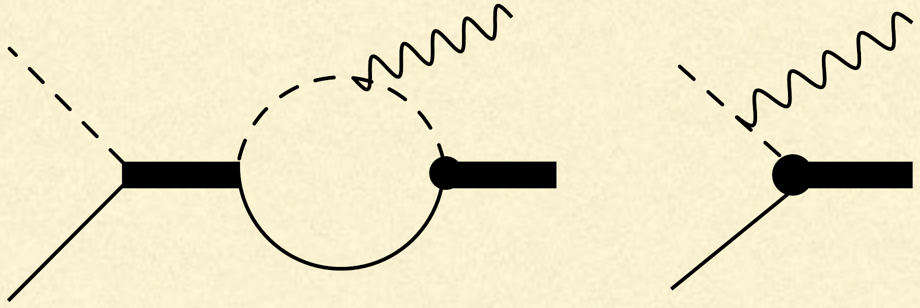
p-wave bound states and capture thereto

Hammer & DP, NPA (2011)

- At LO: p-wave 1n halo described solely by its ANC and binding energy

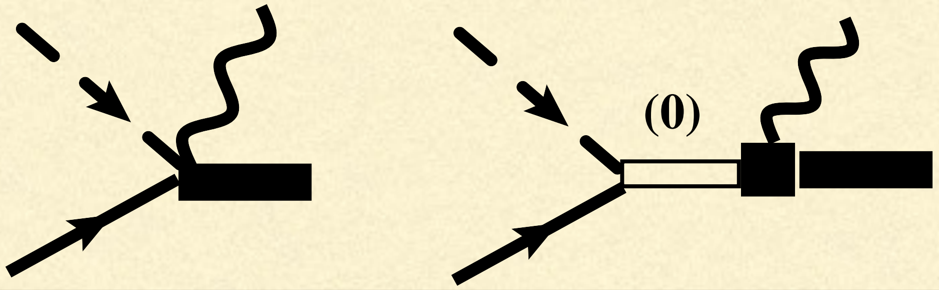
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right) \qquad \gamma_1 = \sqrt{2 m_R B}$$

- Capture to the p-wave state proceeds via the one-body E1 operator: “external direct capture”



$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



^{7}Be + p$\rightarrow$^{8}B + γ_{E1} at LO in Halo EFT

Zhang, Nollett, DP, Phys. Rev. C 89, 051602 (2014);
Ryberg, Forssen, Hammer, Platter, EPJA (2014)

- In this system R_{core}~3 fm, R_{halo} ~15 fm; scale of Coulomb interactions: $k_C=Q_cQ_n\alpha_{EM}M_R$=24 MeV; a~10 fm, both also ~R_{halo}

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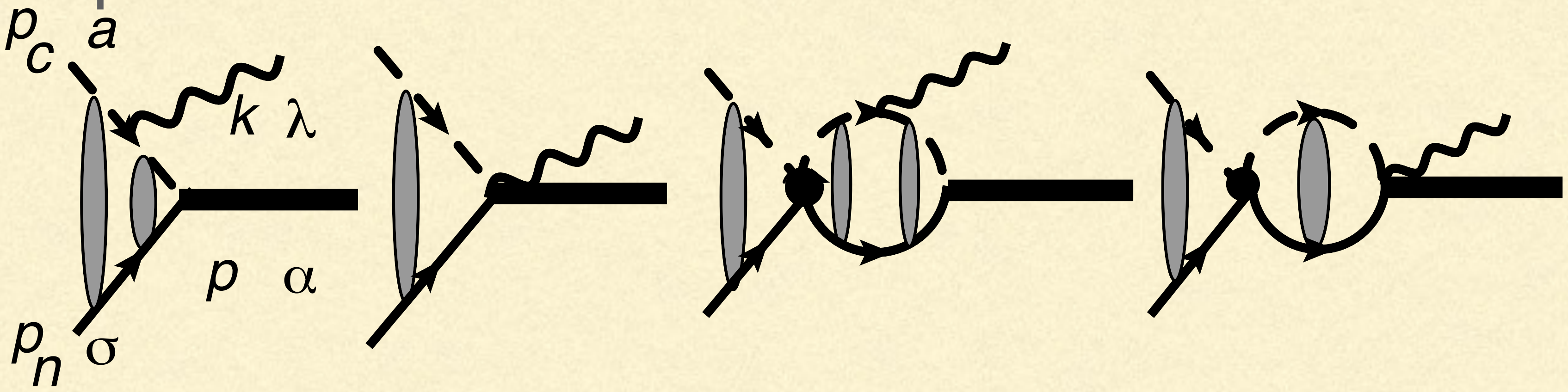

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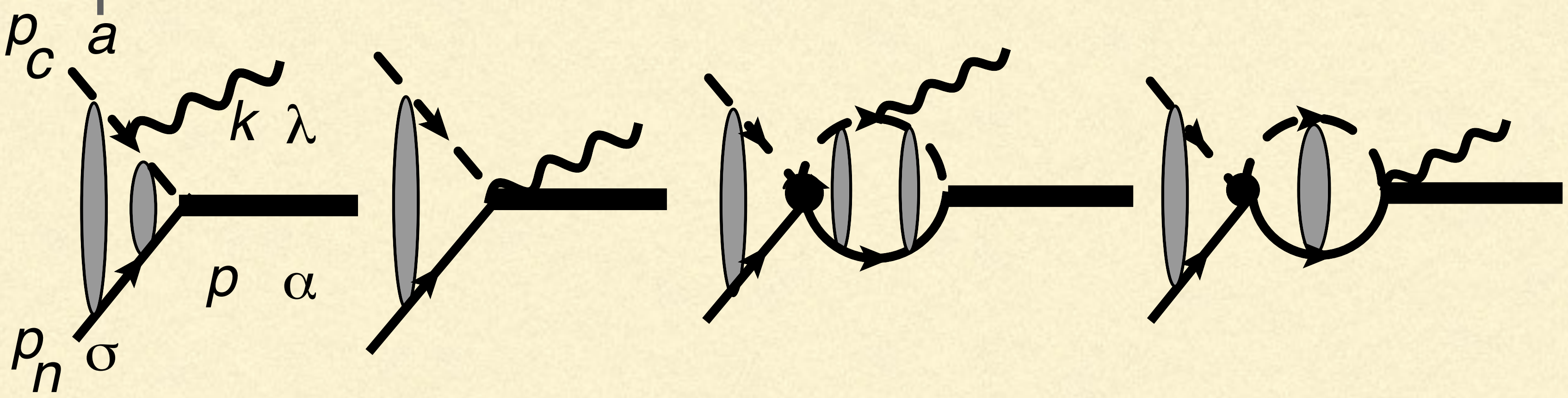

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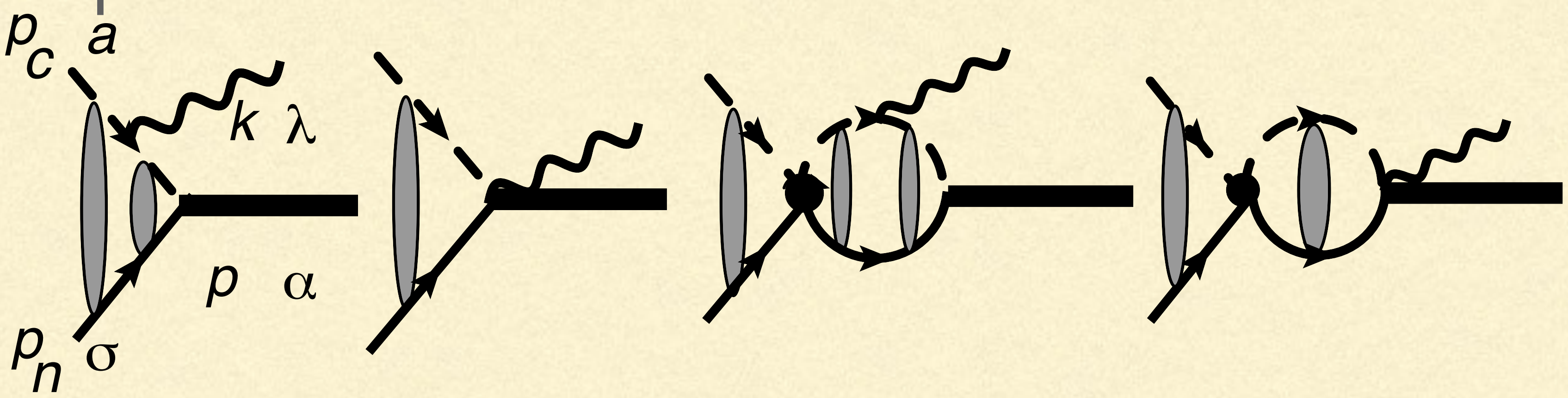

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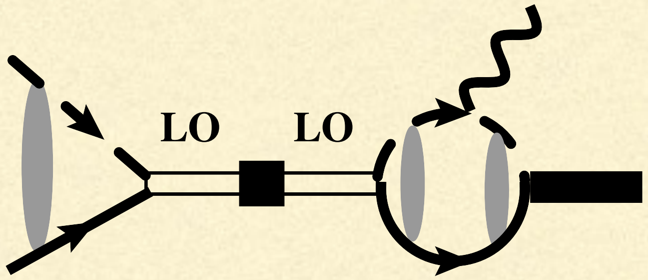
$$S(E) = f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E;\delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

Four parameters
at leading order

Additional ingredients at NLO

Zhang, Nollett, DP, hys. Lett. B751, 535 (2015), arXiv:1708.04017;
Ryberg, Forssen, Platter, Ann. Phys. (2016)



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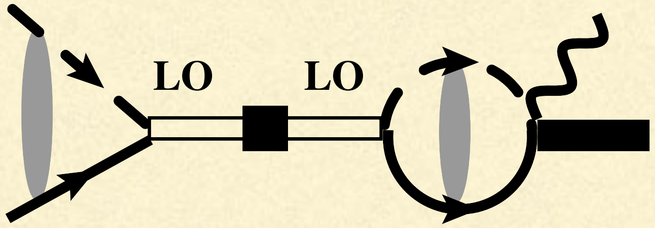

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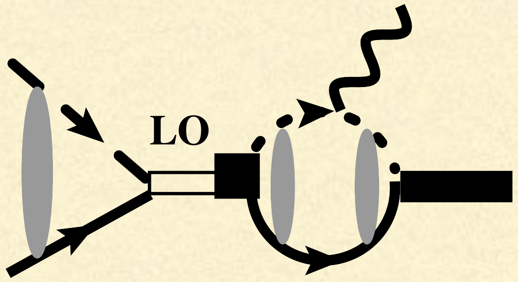

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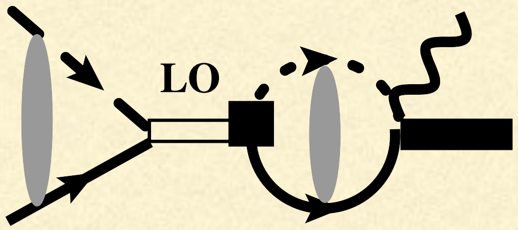

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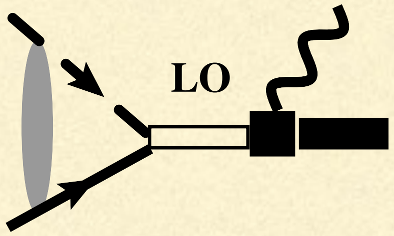

(IX)

$$S(E) = \quad f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E;\delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\rm SD}\left(E;\delta_s(E)\right) + \epsilon_s \mathcal{S}_{\rm CX}\left(E;\delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

- Effective ranges in both $^5S_2$ and $^3S_1$: r_2 and r_1
- Core excitation: determined by ratio of ^{8}B couplings of ^{7}Be*p and ^{7}Be-p states: ϵ_1
- LECs associated with contact interaction, one each for S=1 and S=2: L_1 and L_2

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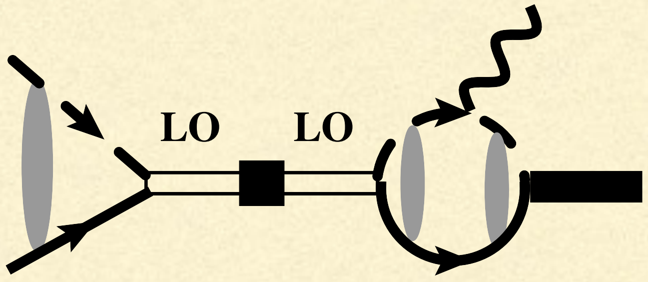
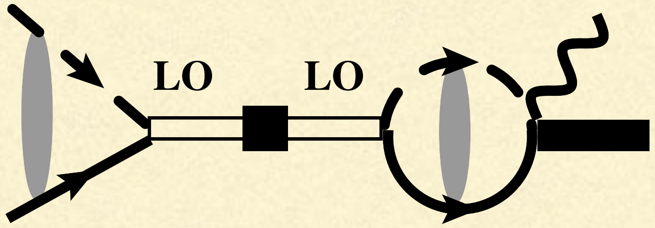
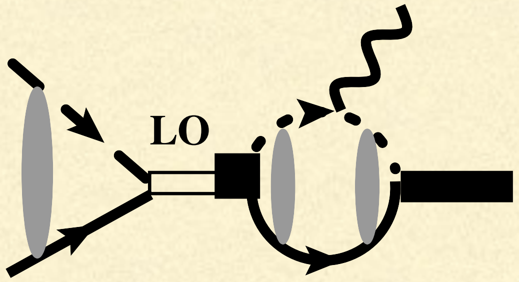


(V)

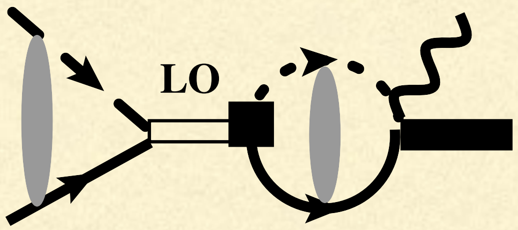


(VI)

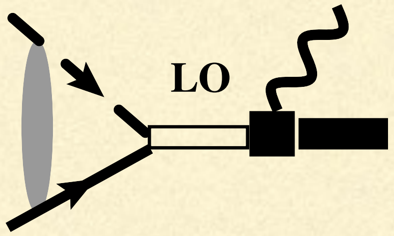
Five more parameters at NLO



(VII)



(VIII)



(IX)

$$S(E) = \quad f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E; \delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\rm SD}\left(E; \delta_s(E)\right) + \epsilon_s \mathcal{S}_{\rm CX}\left(E; \delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

- Effective ranges in both $^5S_2$ and $^3S_1$: r_2 and r_1
- Core excitation: determined by ratio of ^{8}B couplings of ^{7}Be*p and ^{7}Be-p states: ϵ_1
- LECs associated with contact interaction, one each for S=1 and S=2: L_1 and L_2

Additional ingredients at NLO

Zhang, Nollett, DP, hys. Lett. B751, 535 (2015), arXiv:1708.04017;
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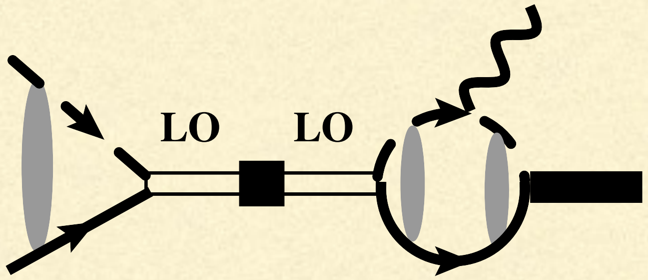
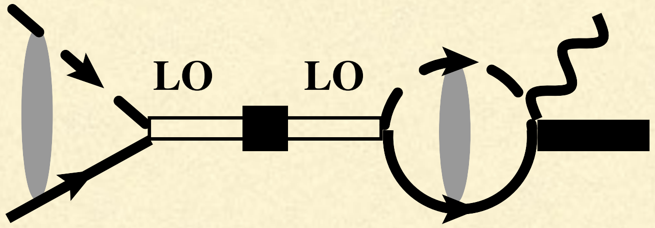


(V)



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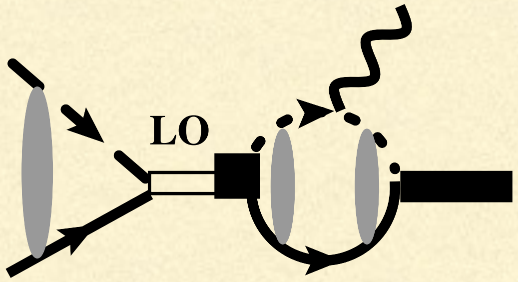
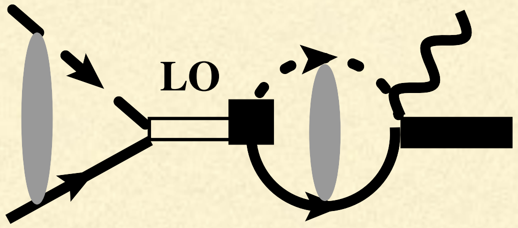
Five more parameters at NLO



(VII)



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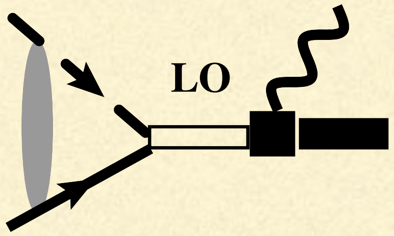

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Data for $^{7}Be + p \rightarrow {}^{8}B + \gamma_{E1}$

- 42 data points for 100 keV < $E_{c.m.}$ < 500 keV
    - Junghans (BE1 and BE3)
    - Fillipone
    - Baby
    - Hammache (1998 and 2001)

Data for ${}^{7}\text{Be} + \text{p} \rightarrow {}^{8}\text{B} + \gamma_{\text{E1}}$

42 data points for 100 keV < $E_{\text{c.m.}}$ < 500 keV	CMEs
Junghans (BE1 and BE3)	2.7% and 2.3%
Fillipone	11.25%
Baby	5%
Hammache (1998 and 2001)	2.2% (1998)

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- CMEs
  - 2.7% and 2.3%
  - 11.25%
  - 5%
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- Subtract M1 resonance: negligible impact at 500 keV and below
- Deal with CMEs by introducing five additional parameters, ξ_i

Building the pdf

$$\mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)\propto \mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)\mathrm{pr}\left(\vec{g},\{\xi_i\}|I\right),$$

$$\ln \mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)=c-\sum_{j=1}^{N}\frac{\left[(1-\xi_j)S(\vec{g};E_j)-D_j\right]^2}{2\sigma_j^2},$$

Building the pdf

- Bayes:

$$\mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right) \propto \mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)\mathrm{pr}\left(\vec{g},\{\xi_i\}|I\right),$$

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- Second factor: priors
  - Independent gaussian priors for ξ_i, centered at zero and with width=CME
  - Gaussian priors for $a_{S=1}$ and $a_{S=2}$, based on Angulo et al. measurement
  - Other EFT parameters, $r_{S=1}$, $r_{S=2}$, L_1, L_2, ANCs, ϵ_1, assigned flat priors, corresponding to natural ranges
  - No s-wave resonance below 600 keV

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Extrinsic information

  - Other EFT parameters, $r_{S=1,}$ $r_{S=2}$, L_1, L_2, ANCs, ϵ_1, assigned flat priors, corresponding to natural ranges
  - No s-wave resonance below 600 keV

Outputs and lessons

- Posteriors on parameters tell us about physics: which combinations are actually constrained?
- How do we see when parameters are not well constrained?
- Extrapolation
- Does EFT truncation error at NLO affect the answer?
- Feedback with experiment: systematic errors? Future experiments?

Posterior plots⇒Physics

$$\mathrm{pr}\left(g_1, g_2|D;T;I\right) = \int \mathrm{pr}\left(\vec{g}, \{\xi_i\}|D;T;I\right)\, d\xi_1 \ldots d\xi_5 dg_3 \ldots dg_9$$

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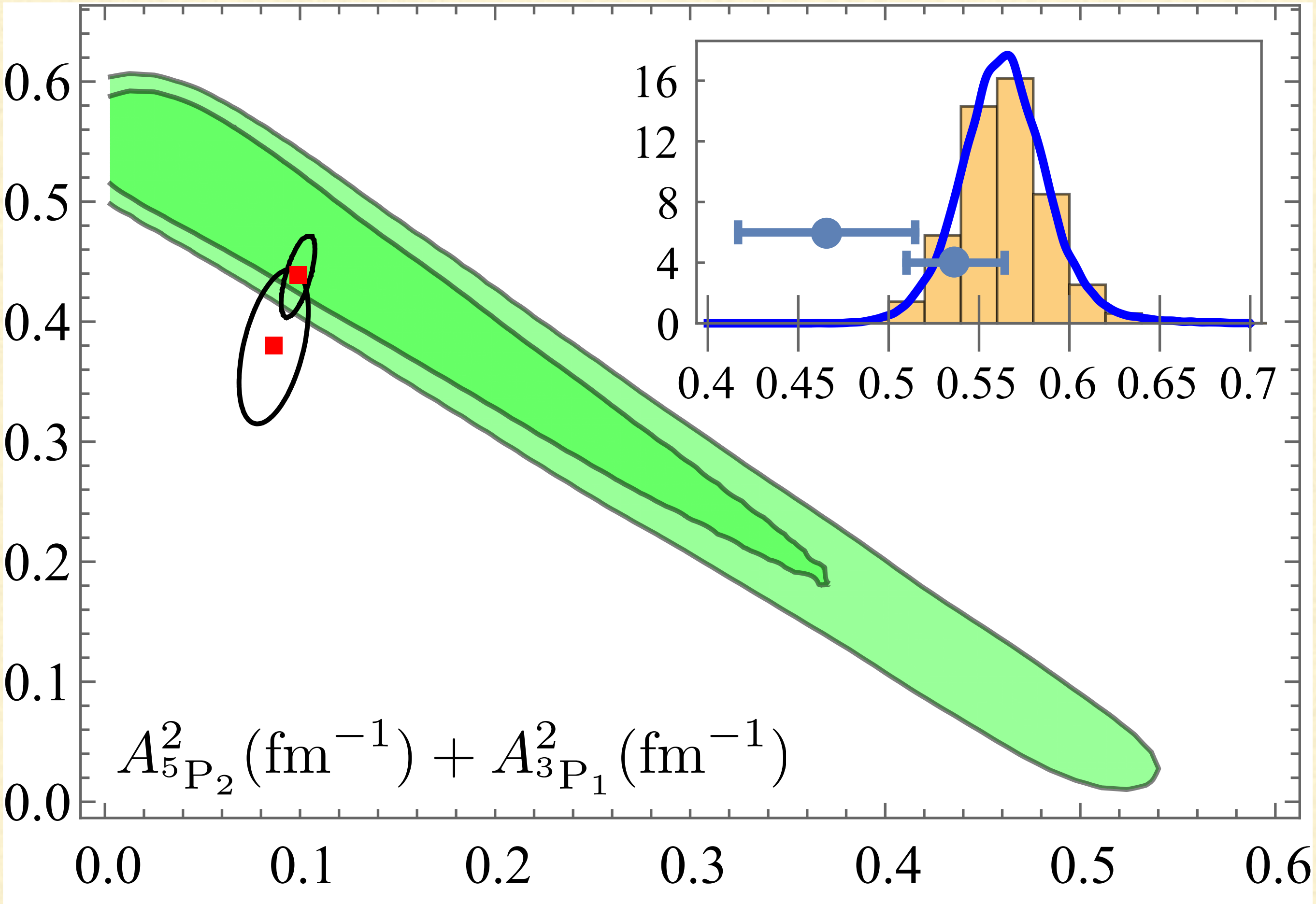

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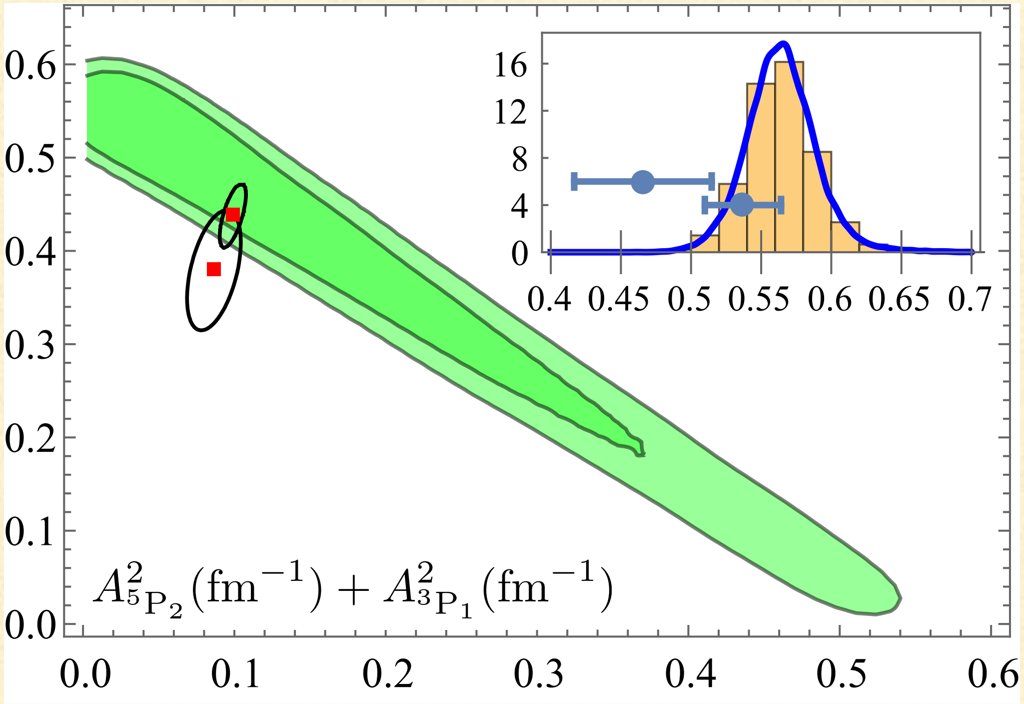
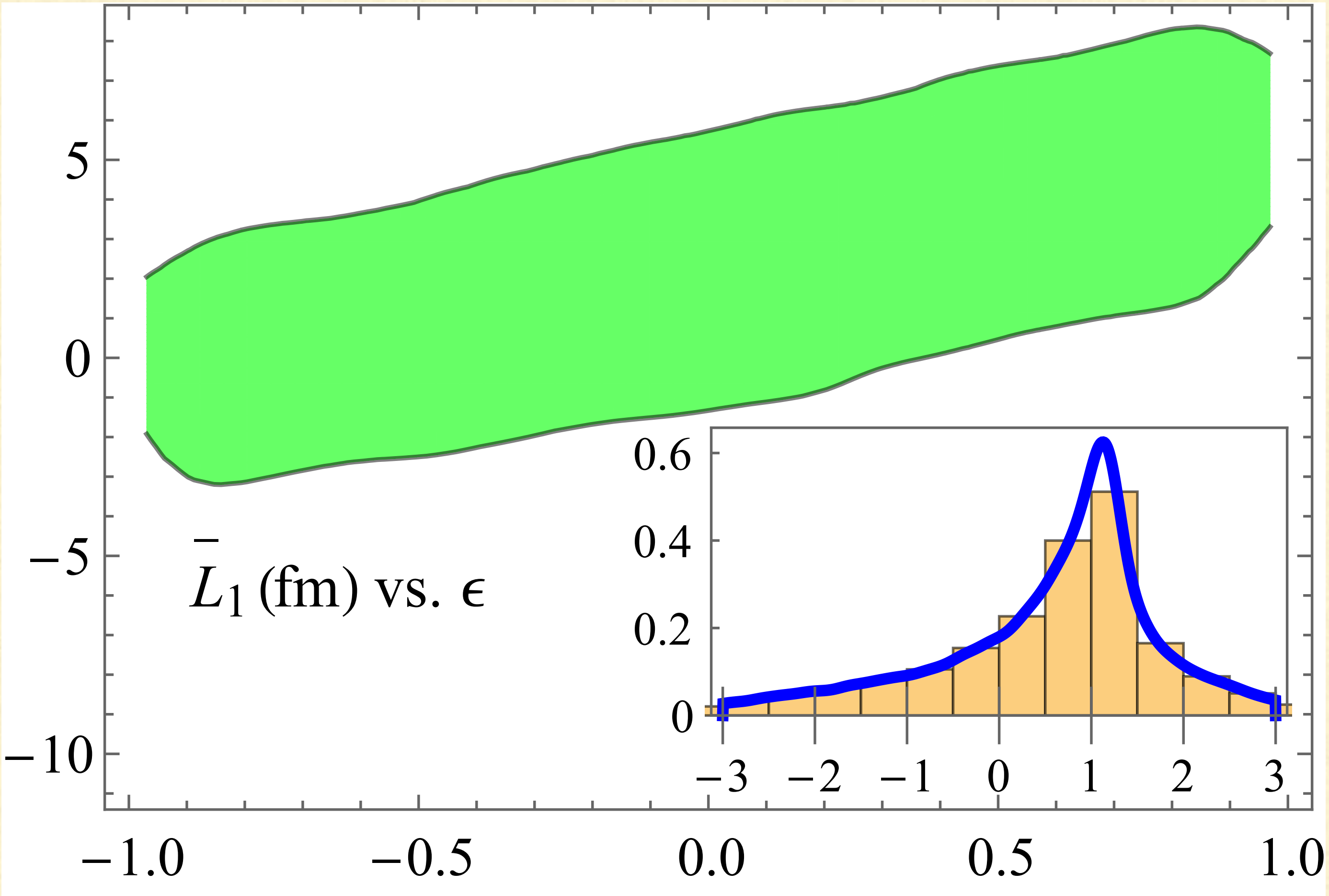
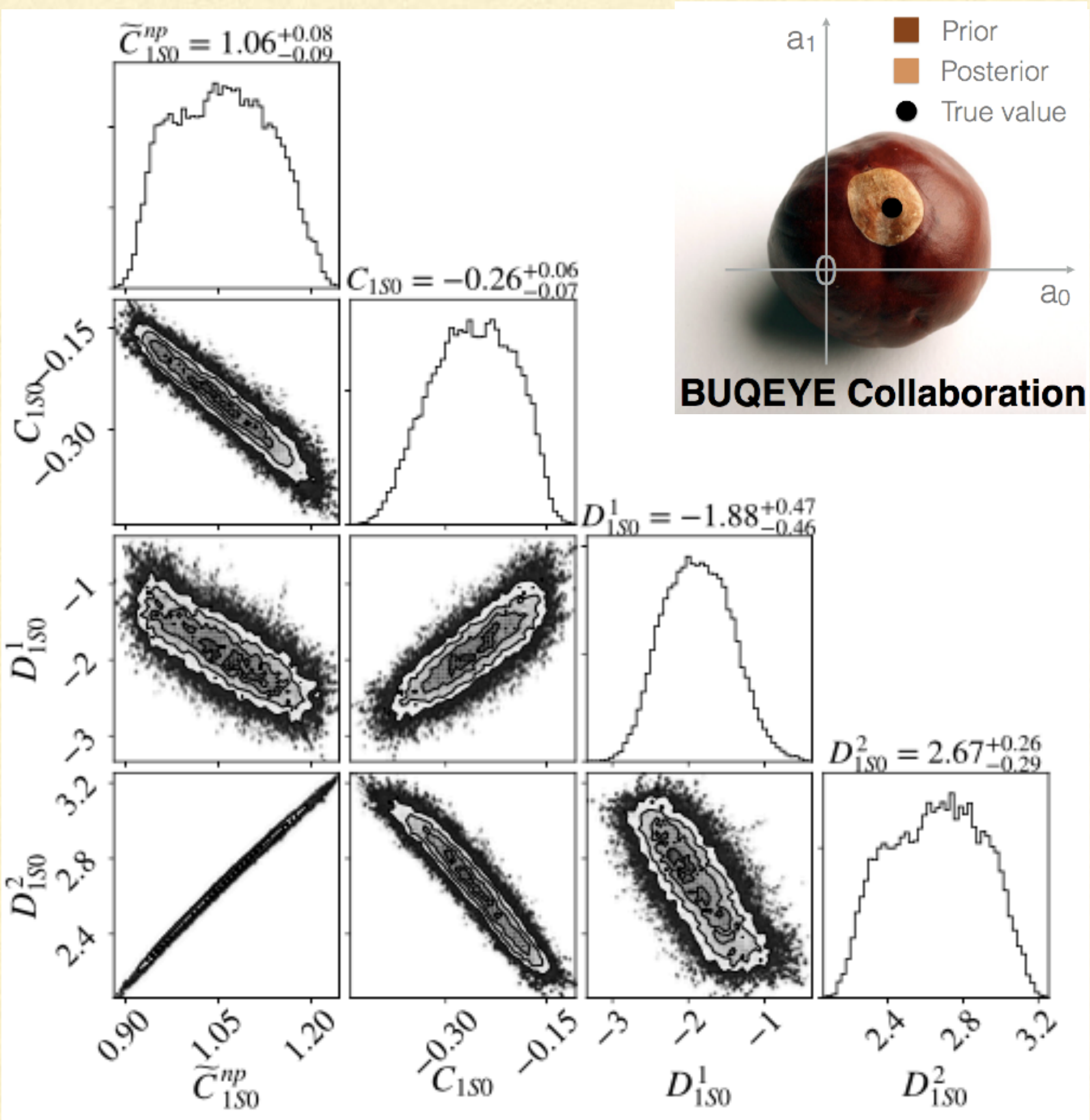




- ANCs are highly correlated but sum of squares strongly constrained
- One spin-1 short-distance parameter: $0.33\ \bar{L}_1/(\mathrm{fm}^{-1}) - \epsilon_1$

Another example of posterior plots



Wesolowski, Furnstahl, DP, in preparation

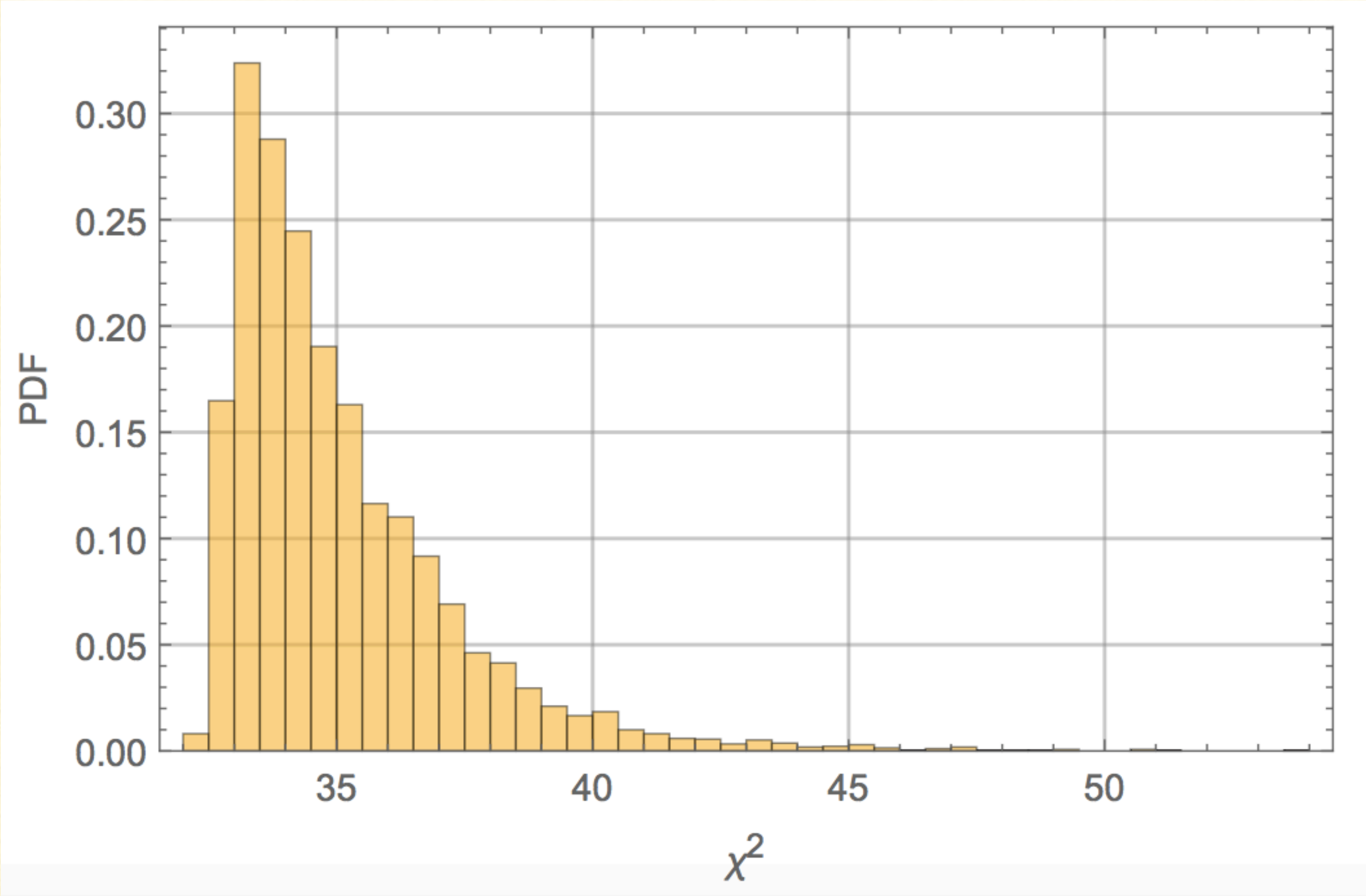
- Parameter estimation for a particular piece of the NN potential at N3LO in the chiral EFT expansion
- Posterior plot allows diagnosis of parameter degeneracy $D^1_{(1S0)}$-$D^2_{(1S0)}$
- Which we also understand analytically

On-shell

$$\langle {}^1S_0|V_{NN}|{}^1S_0\rangle = D^1_{(1S0)}\,p^2p'^2 + D^2_{(1S0)}\,(p^4+p'^4)$$

$$= \frac{1}{4}(D^1_{(1S0)}+2D^2_{(1S0)})(p^2+p'^2)^2 - \cancelto{0}{\frac{1}{4}(D^1_{(1S0)}-2D^2_{(1S0)})(p^2-p'^2)^2}\,,$$

More questions we can answer



42 data points,
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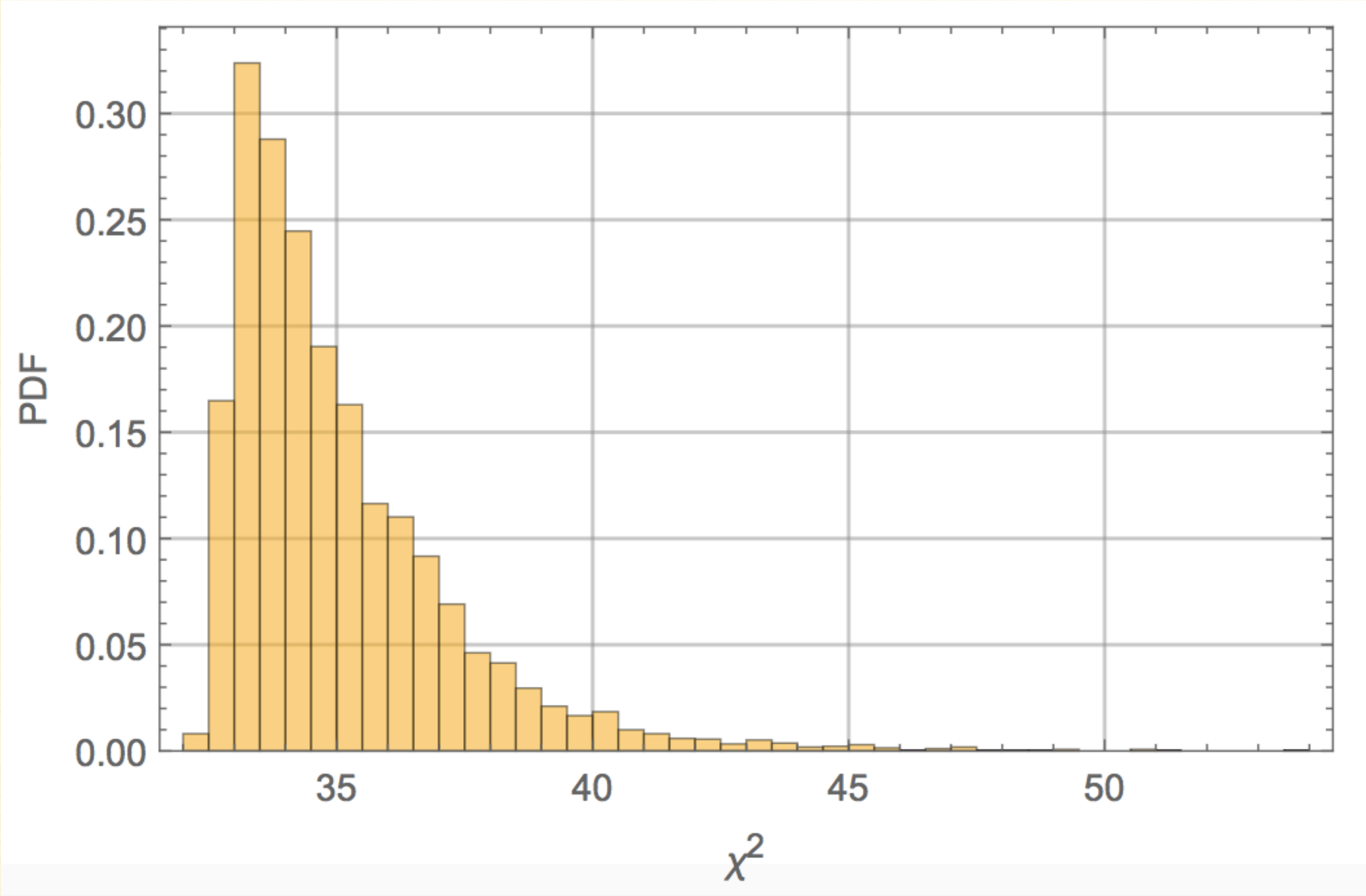

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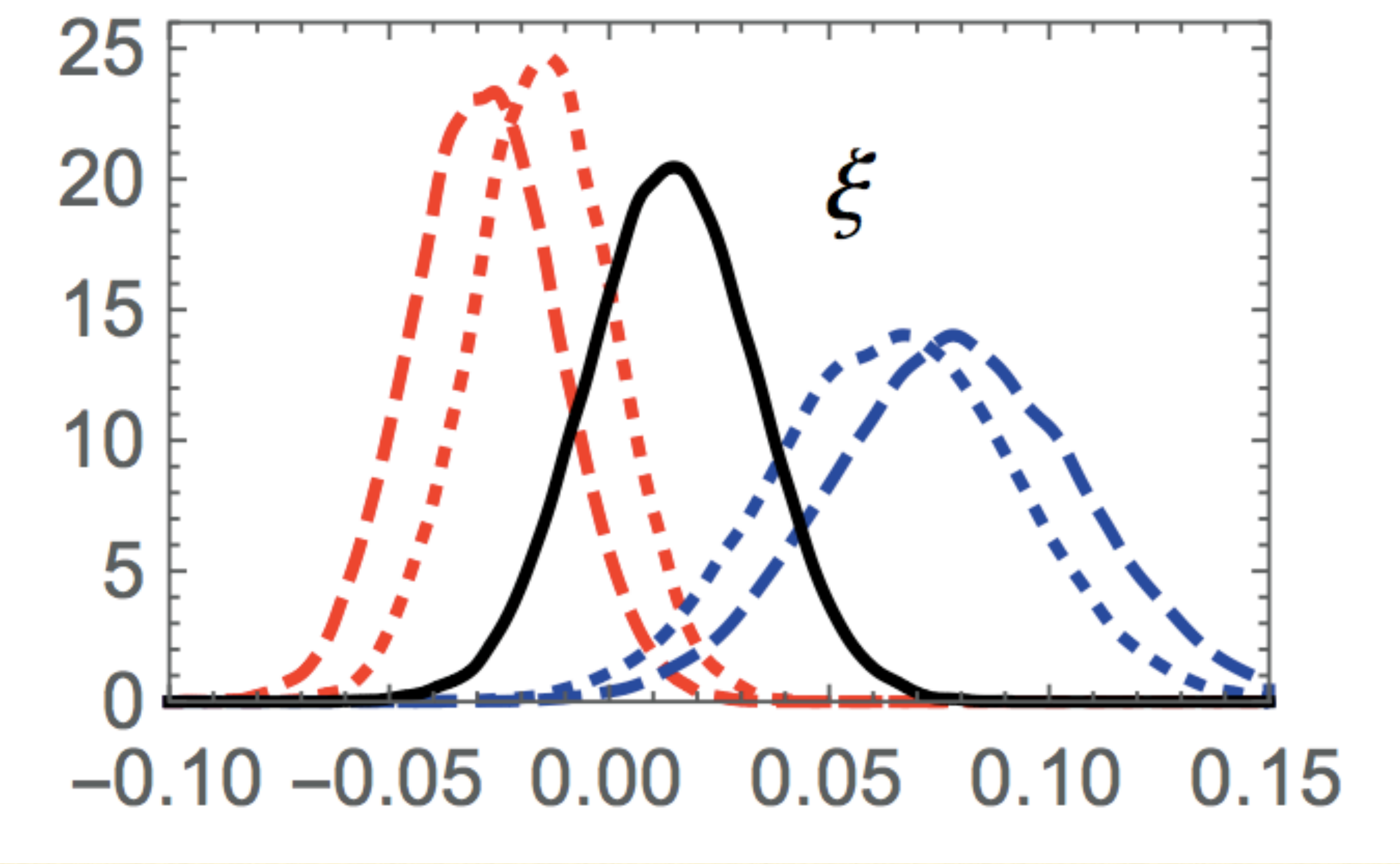
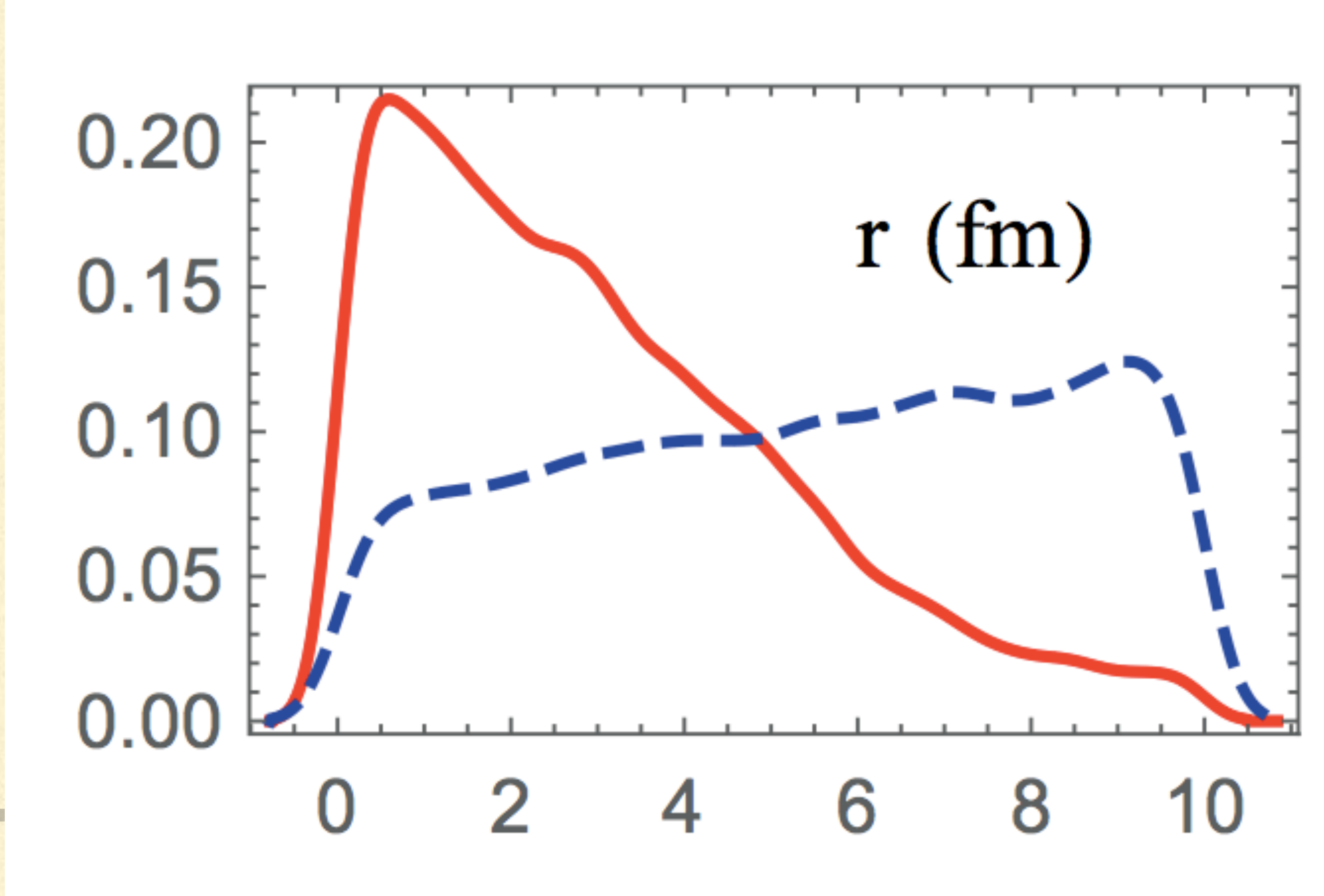


More questions we can answer

- Is it a "good fit"?
- Did the experimentalists understand their systematic errors?
- Are there parameters that are not well constrained by these data?



Final result

Zhang, Nollett, DP, PLB, 2015

$$\mathrm{pr}\left(\bar{F}|D;T;I\right) = \int \mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)\delta(\bar{F} - F(\vec{g}))d\xi_1 \ldots d\xi_5 d\vec{g}$$

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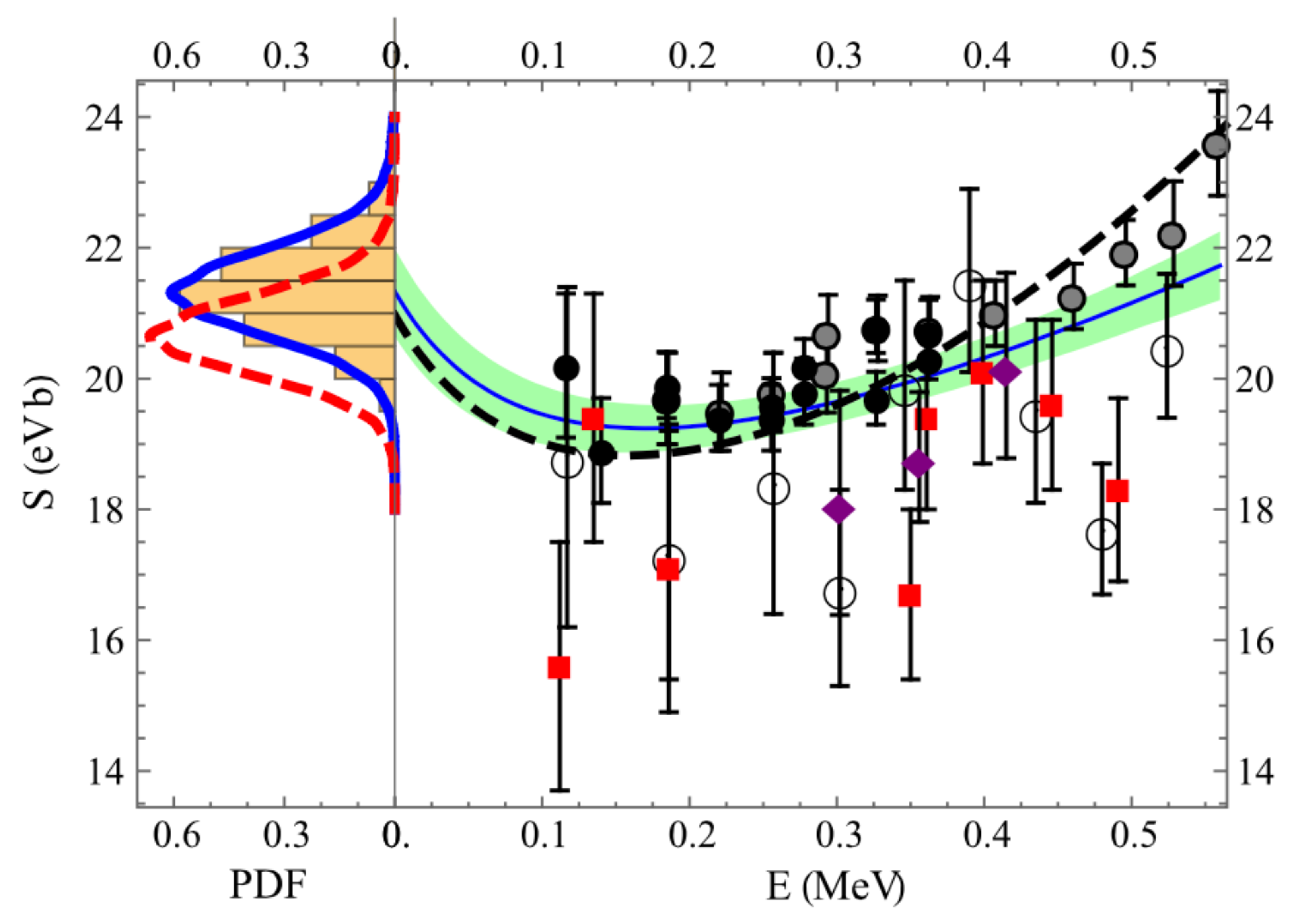

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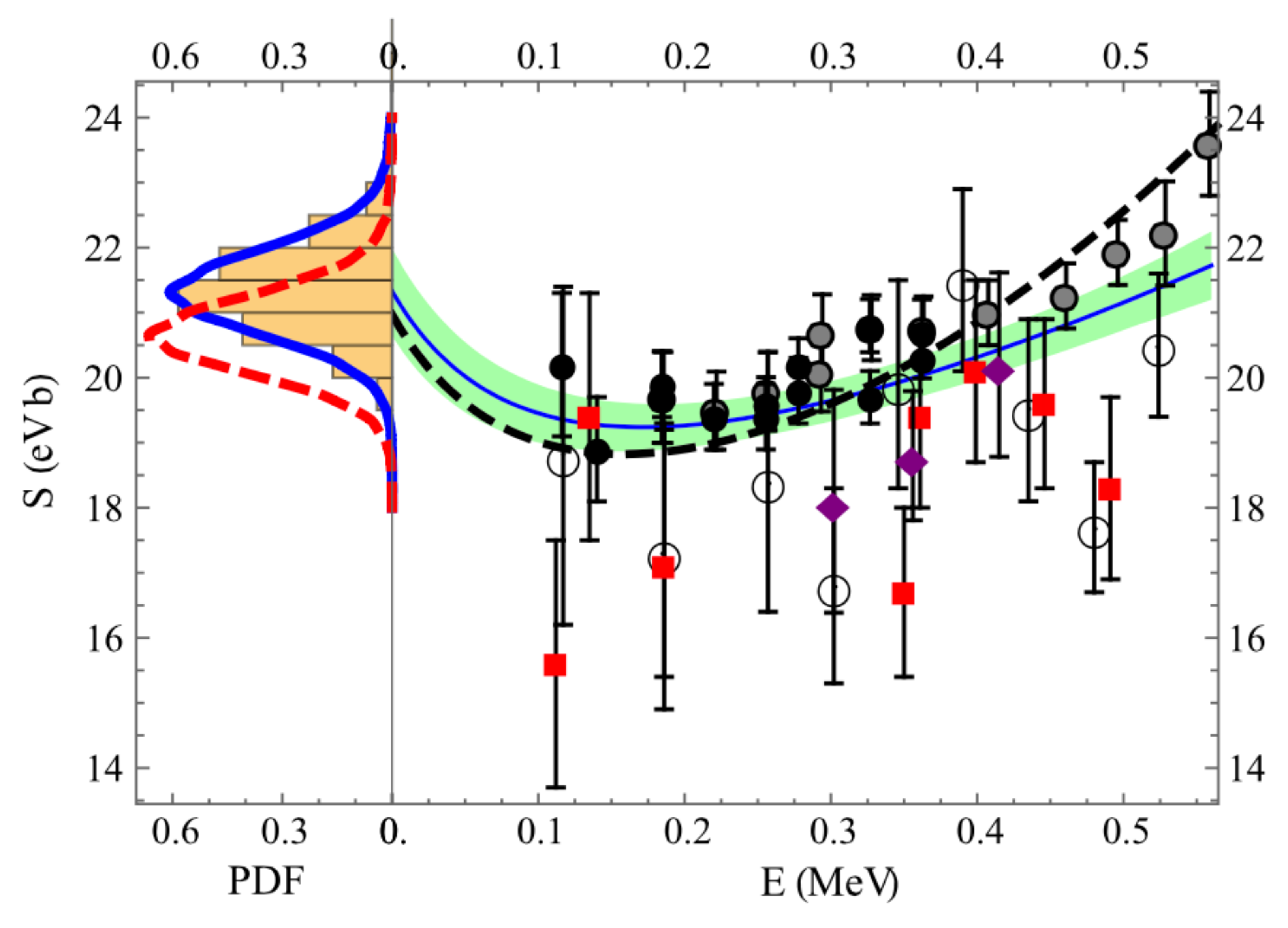

$$S(0) = 21.33^{+0.66}_{-0.69}\ \mathrm{eV\ b}$$

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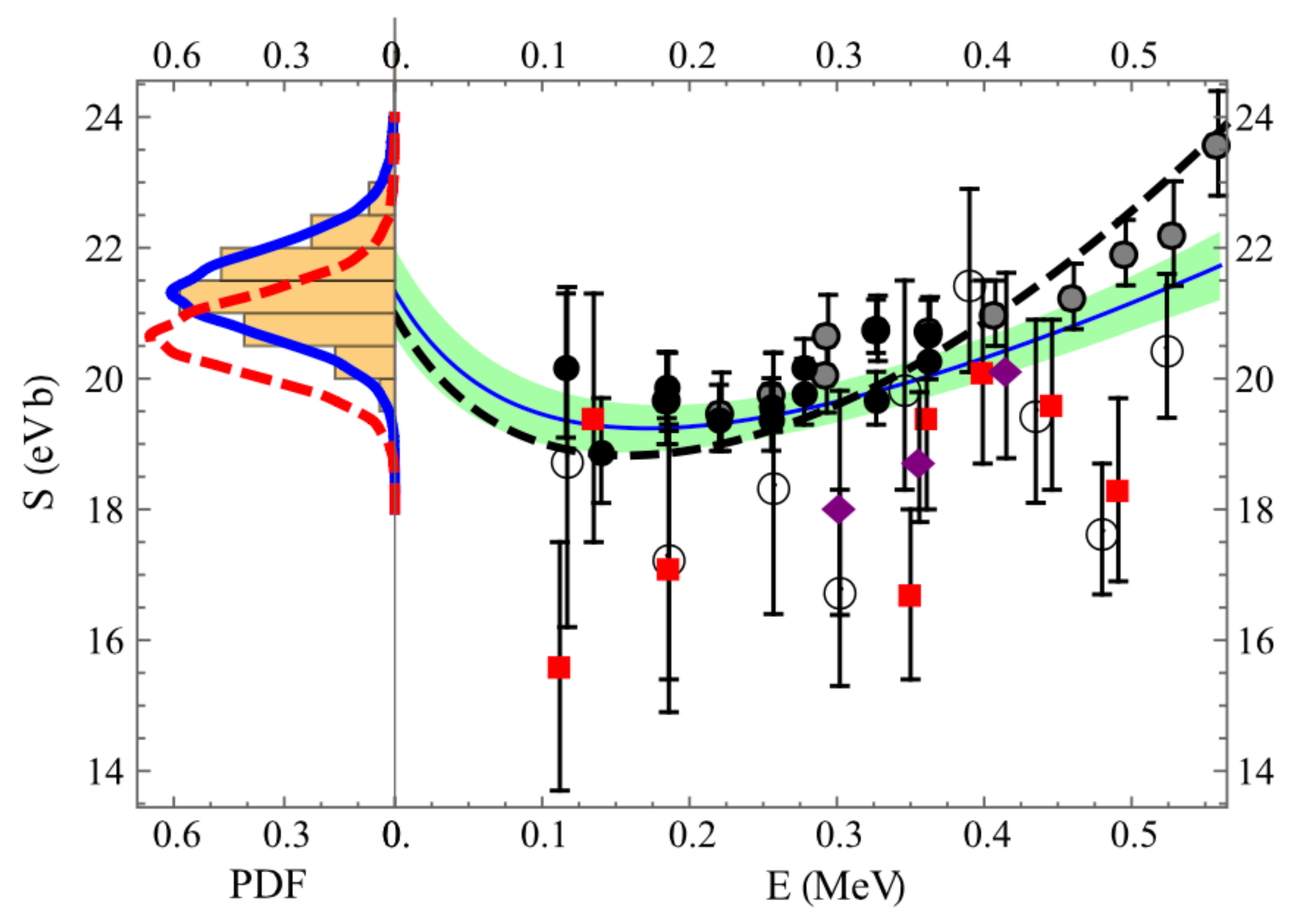

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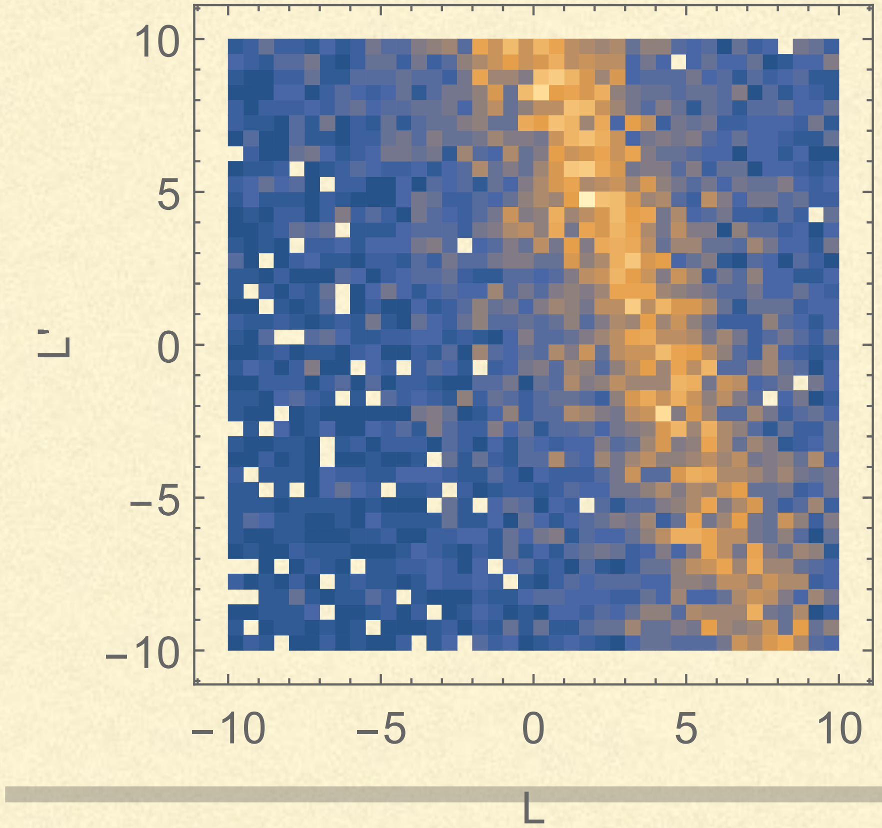
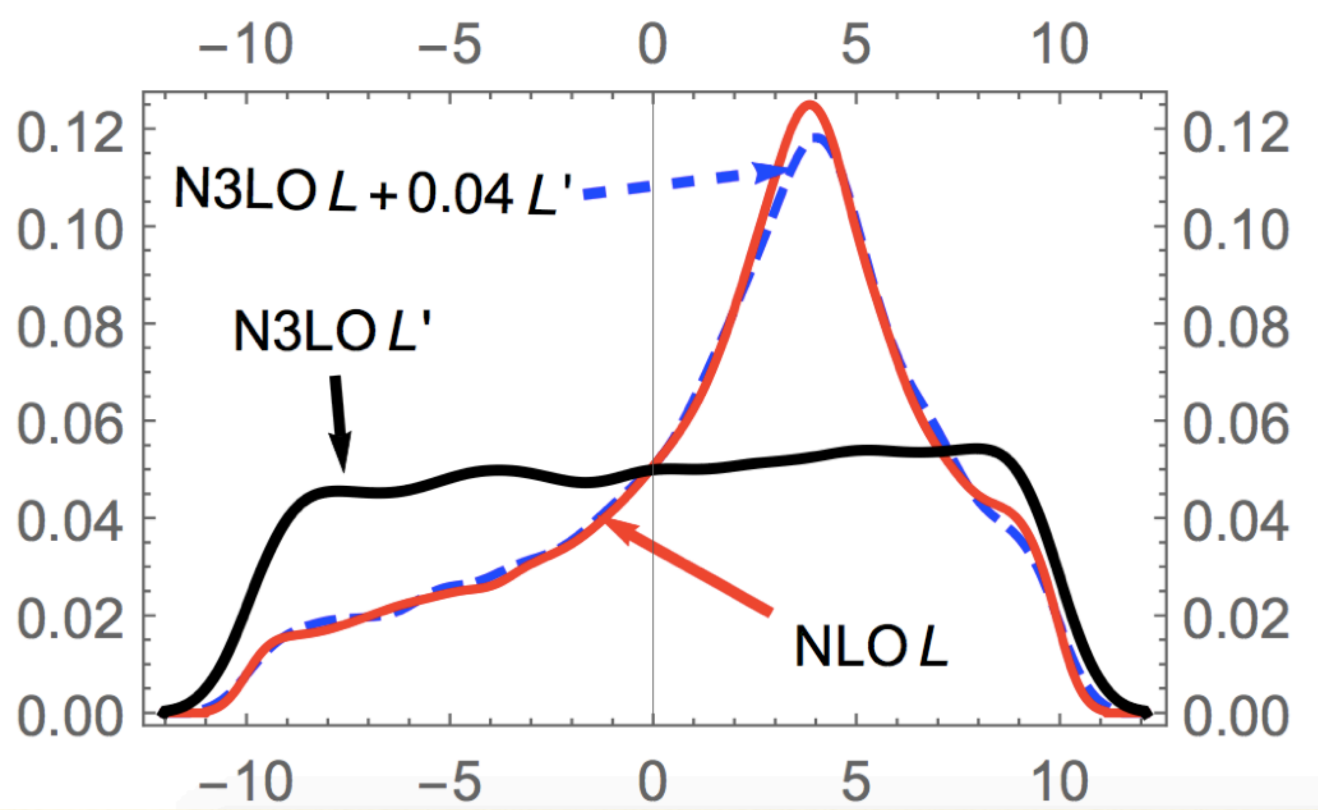
Uncertainty reduced by factor of two: model selection

Truncation error

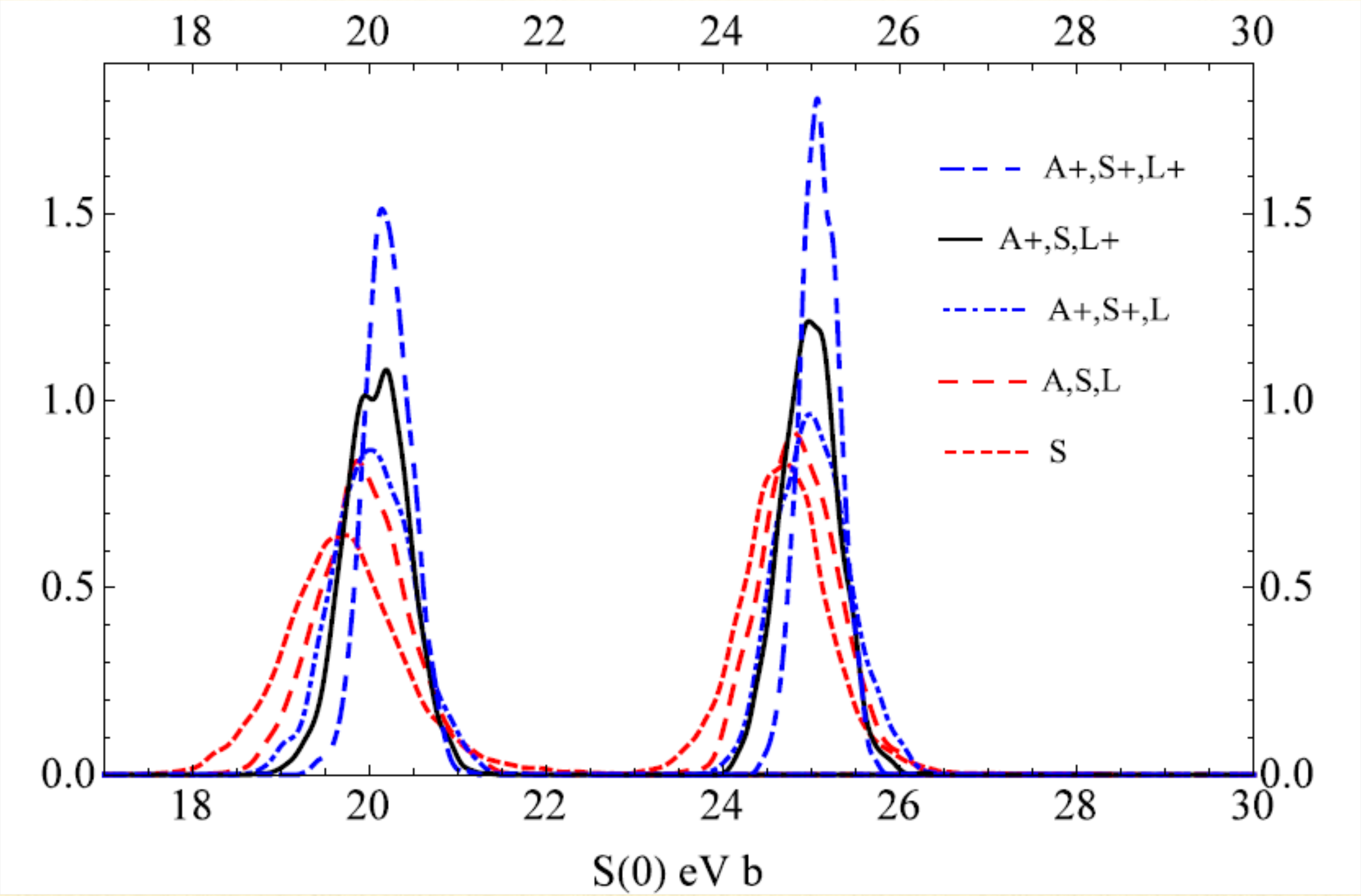
- N2LO correction=0 (technically only in absence of excited state)
- EFT s-wave scattering corrections (shape parameter)~0.8%
- E2, M1 contributions < 0.01%, Radiative corrections: ~0.1%
- So first correction is at N3LO, i.e., $\overline{L}_i \to \overline{L}_i + k^2\overline{L}_i'$





Planning improvements

Use extrapolant to simulate impact of hypothetical future data that could inform posterior pdf for S(0)



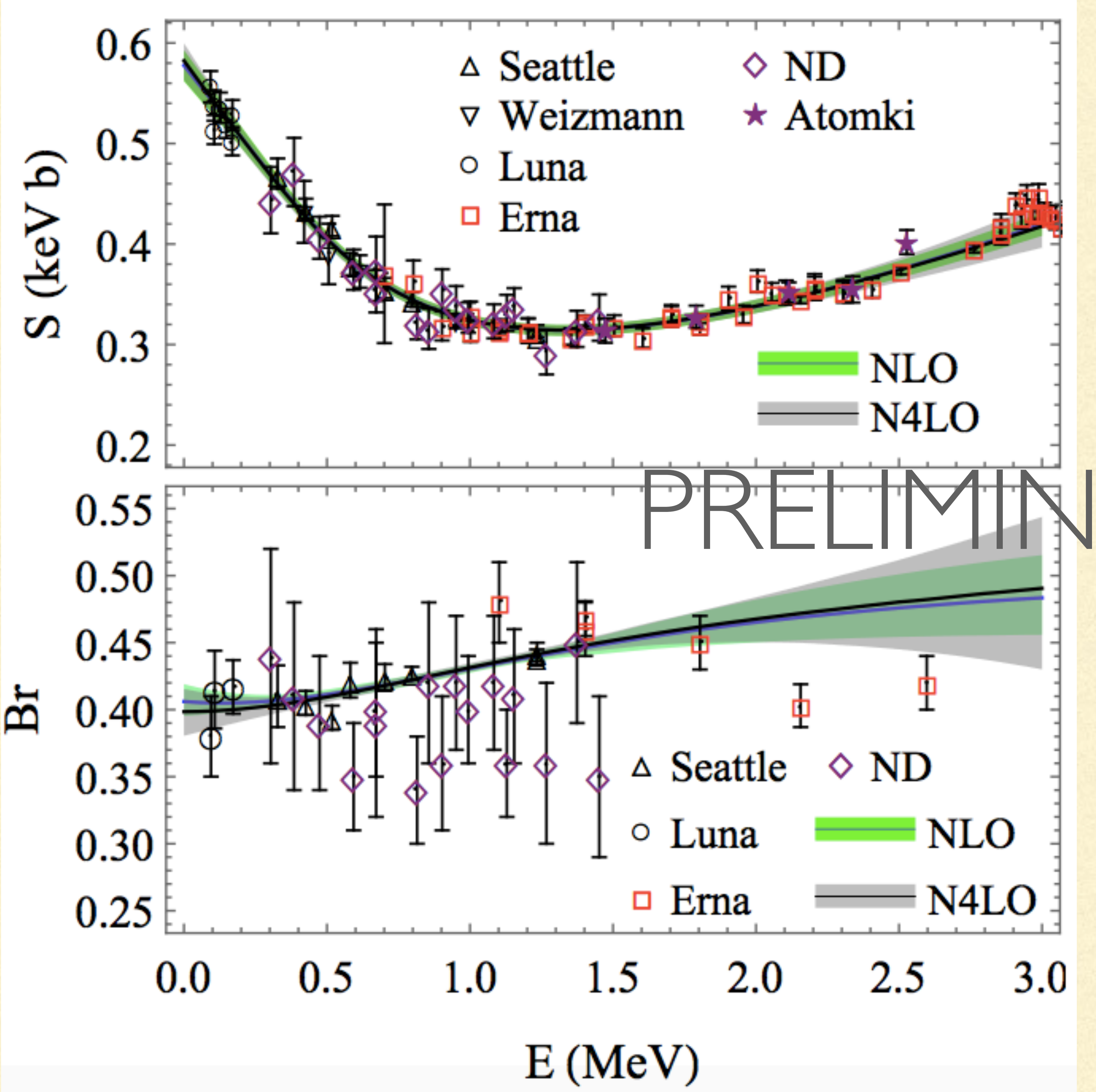
Left-to-right:
42 data points all of similar quality to Junghans et al.

A: ANC
S: $a_{S=1}$ and $a_{S=2}$
L: short-distance

Note that 1 keV uncertainty in S_{1p} of ^{8}B may not be negligible effect

A sneak peek at ^{3}He(^{4}He,ɣ)

Zhang, Nollett, DP, in preparation



PRELIMINARY

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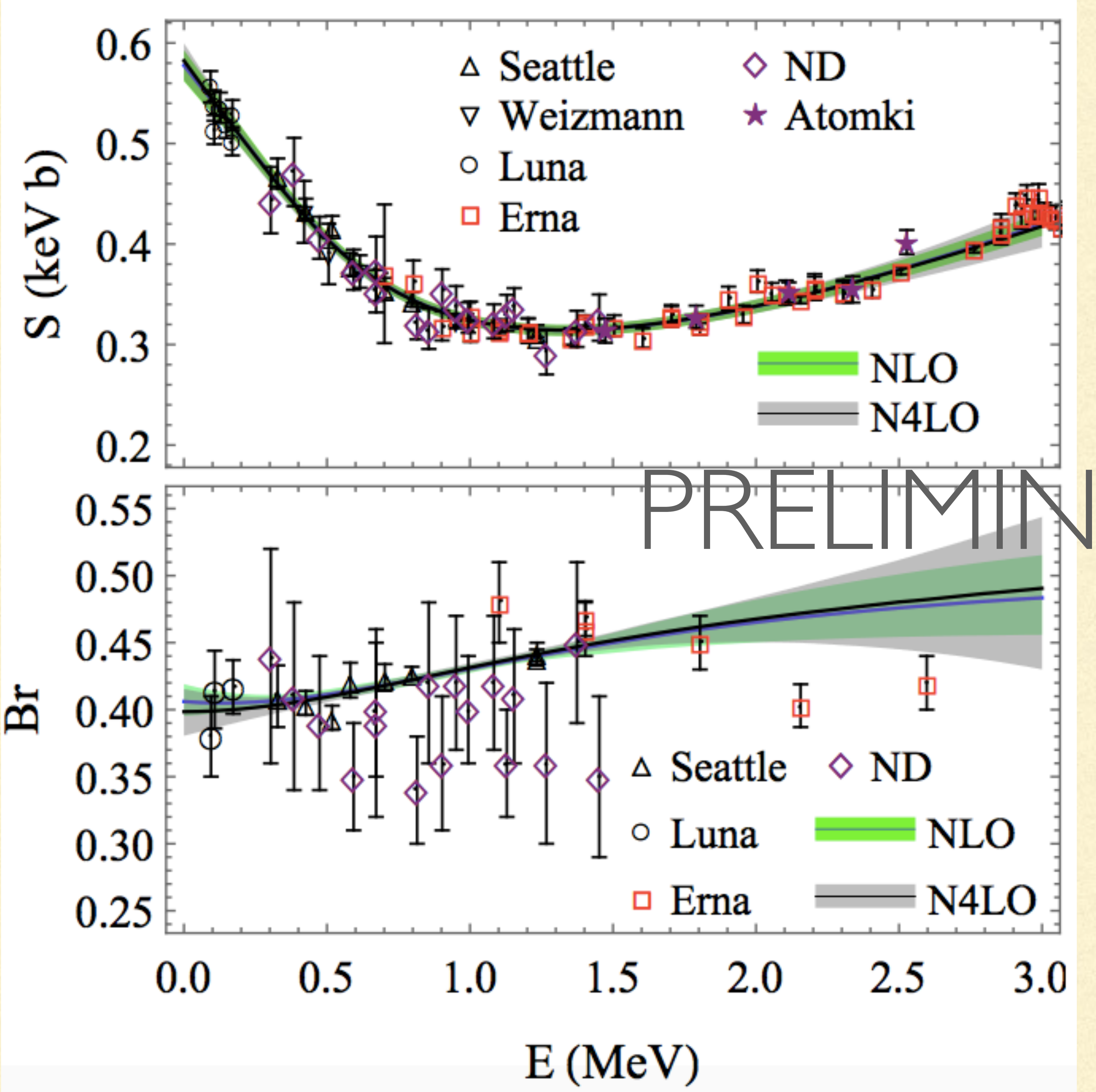
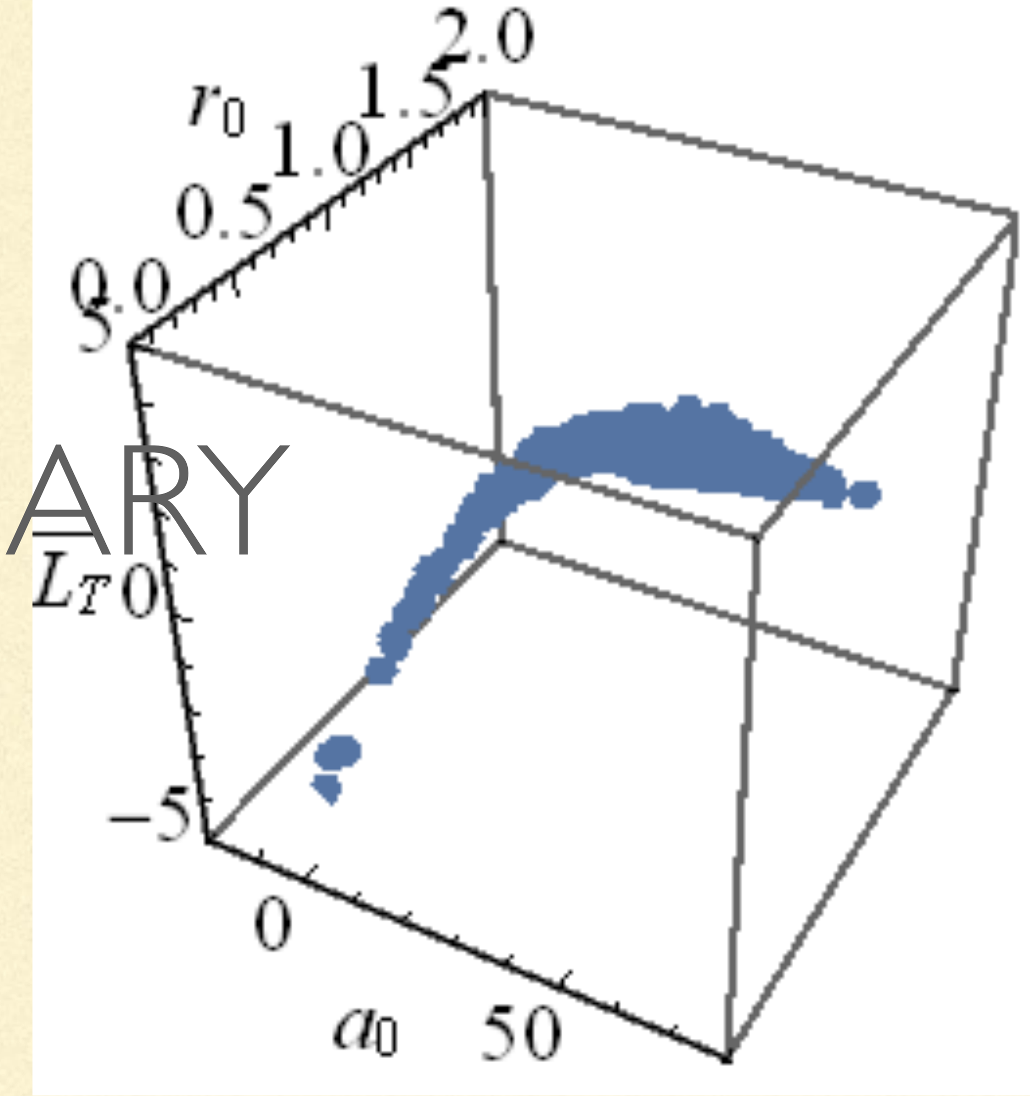
Zhang, Nollett, DP, in preparation



PRELIMINARY



Summary

- EFT provides following features for capture reactions
    - Separation of long- and short-distance dynamics
    - Model-independent (and in two-body case) analytic form for S(E)
    - Ability to reproduce "reasonable models"
- Extrapolation problem formulated as a marginalization over models

$$\mathrm{pr}(S(0)|\mathrm{data}, I) = \int d\mathrm{models}\, \mathrm{pr}(S(0)|\mathrm{model}, I)\, \mathrm{pr}(\mathrm{model}|\mathrm{data}, I)$$

- Taking a variety of "reasonable models" and using them to extrapolate may **over**estimate the model uncertainty
- Application of Halo EFT to ^{7}Be(p,γ)^{8}B produces new S(0), consistent with SFII, but with factor two smaller uncertainty

$$S(0) = 21.33^{+0.66}_{-0.69}\ \mathrm{eV\ b}$$

Stuff I learnt from this study

- Precise extrapolation can be done even when you don't have 10*n data
- Model uncertainty can be accommodated, and standard methods may over-estimate it. But it helps to be dong EFT…
- Priors ultimately diagnosable: unconstrained parameters return the prior, and the results we looked at were not sensitive to different choices of prior. "Robust Bayesian Analysis"?
- Projected posterior reveals which combinations of parameters are constrained/affect this observable
- Truncation errors can be assessed
- Future experiments can be planned for maximum impact

Extensions, references

- Simultaneous fit to 7Be p scattering data: requires inclusion of resonances; "Hierarchical Bayes"
- Coulomb dissociation data?
- Same techniques applied to ^{3}He(^{4}He,γ)

Higa, Rupak, Vaghani, arXiv:1612.08959

- Other, and more sophisticated, examples of Bayesian Uncertainty Quantification, see BUQEYE collaboration papers
  - Quantifying uncertainties due to omitted higher-order terms
  - Bayesian parameter estimation

Fursntahl, Klco, DP, Wesolowki, PRC 92, 024005 (2015)

Melendez, Furnstahl, Wesolowski, arXiv:1704.03308

Wesolowski, Klco, Furnstahl, DP, Thapaliya, JPG 43, 074001 (2016)

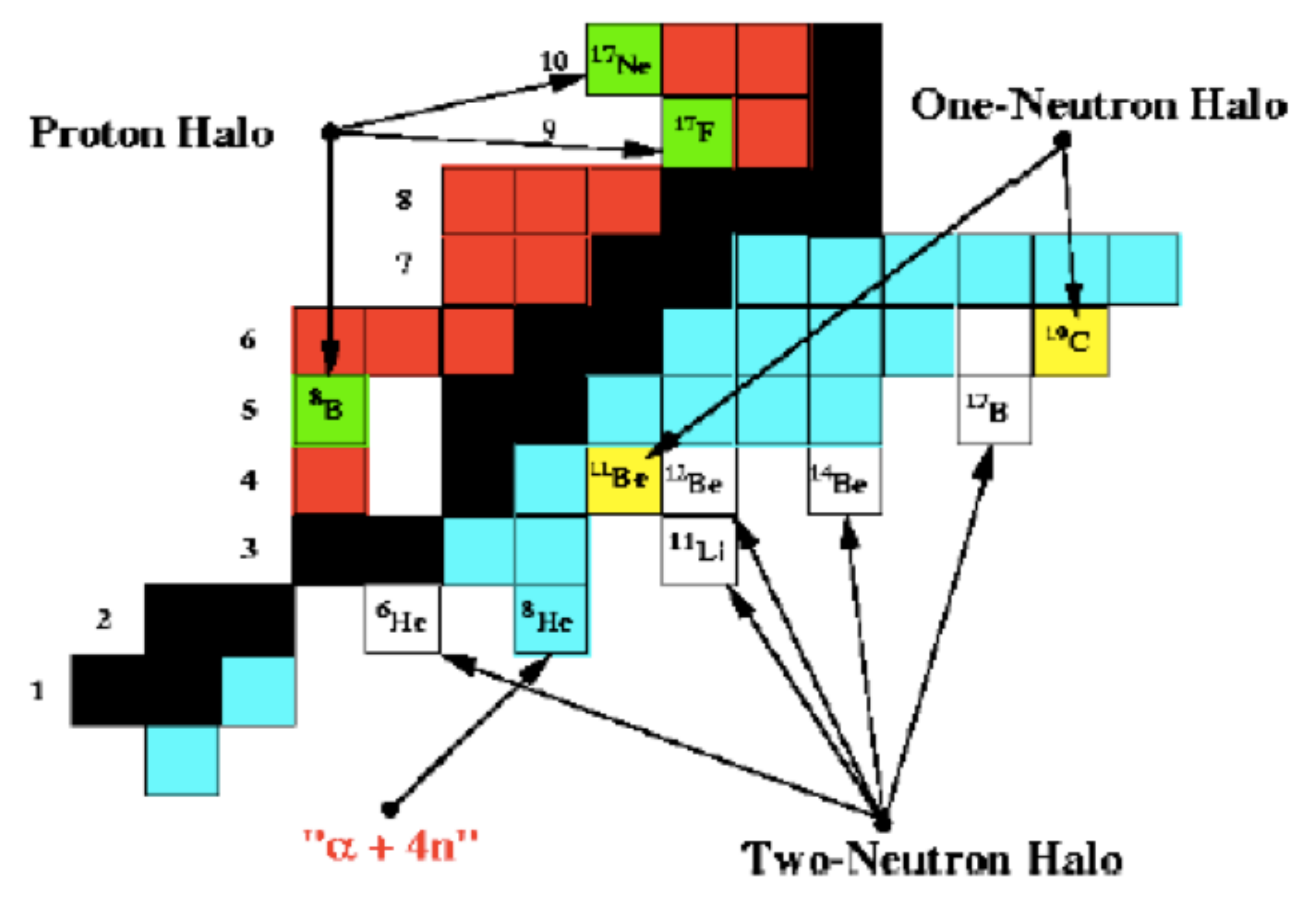
- Review of Halo EFT

Hammer, Ji, DP, JPG 44, 103002 (2017)

Backup Slides

Halo nuclei

http://nupecc.org



Halo nuclei

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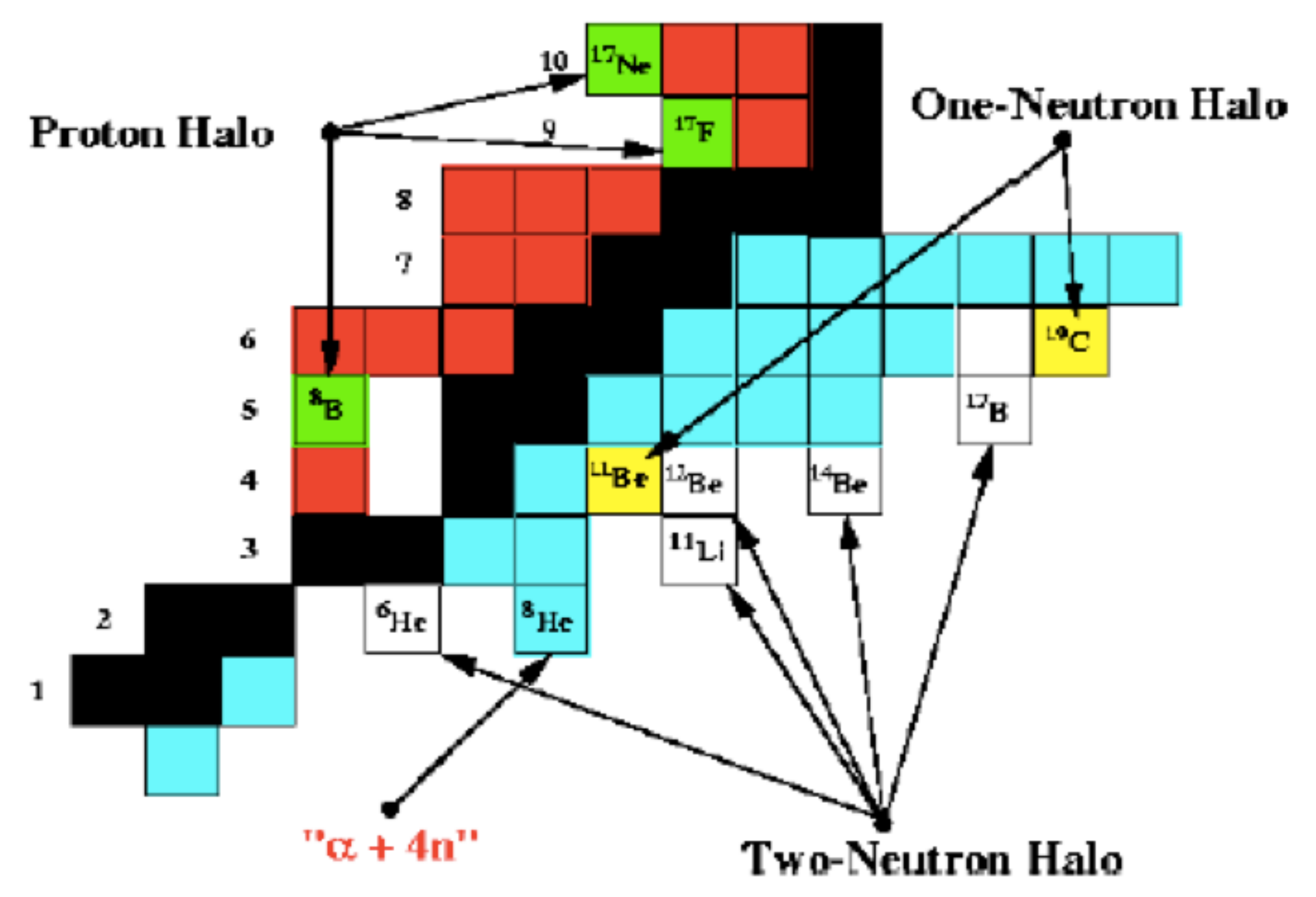

- A halo nucleus as one in which a few (1, 2, 3, 4, ...) nucleons live at a significant distance from a nuclear core.

Halo nuclei

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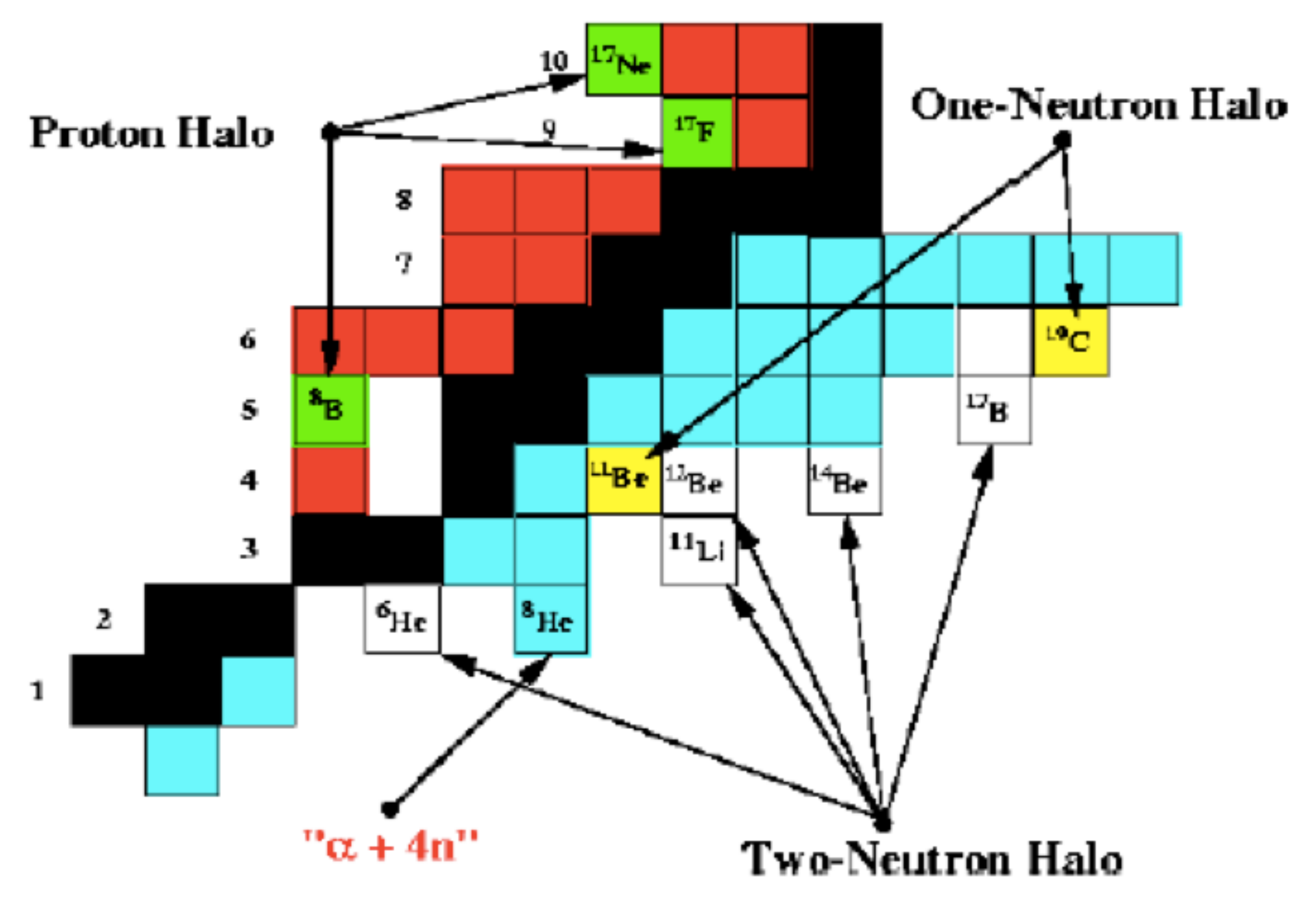

- A halo nucleus as one in which a few (1, 2, 3, 4, ...) nucleons live at a significant distance from a nuclear core.
- Halo nuclei are characterized by small nucleon binding energies, large interaction cross sections, large radii, large E1 transition strengths.

What it does and doesn’t do

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It doesn’t:

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- Need or discuss spectroscopic factors

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It does:

- Connect structure and reactions, including in multi-nucleon halos
- Collect information from different theories/experiments in one calculation
- Treat same physics as cluster models, in a systematically improvable way
- Provide information on inter-dependencies of low-energy observables, including along the core + n, core + 2n, core + 3n, etc. chain

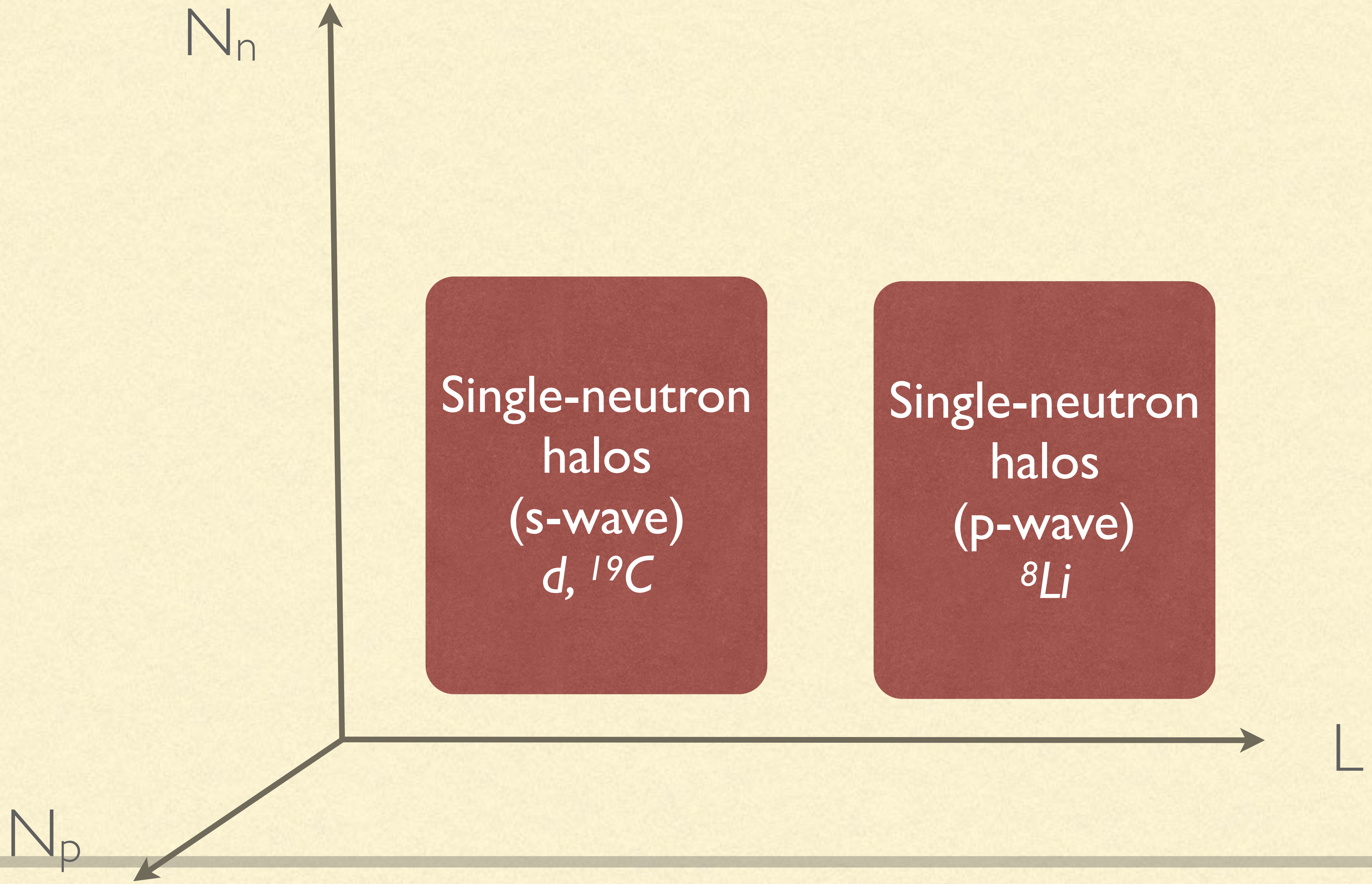
Our approach

- S-wave (and P-wave) states generated by cn contact interactions
- No discussion of nodes, details of n-core interaction, spectroscopic factors

$$u_0(r) = A_0 \exp(-\gamma_0 r)$$

- ^{19}C: input at LO: neutron separation energy of s-wave state.
- A_0 (“wave-function renormalization”) can be fit at NLO.
- P-wave states require two inputs already at LO.

The multi-dimensional Halo EFT space



N_n

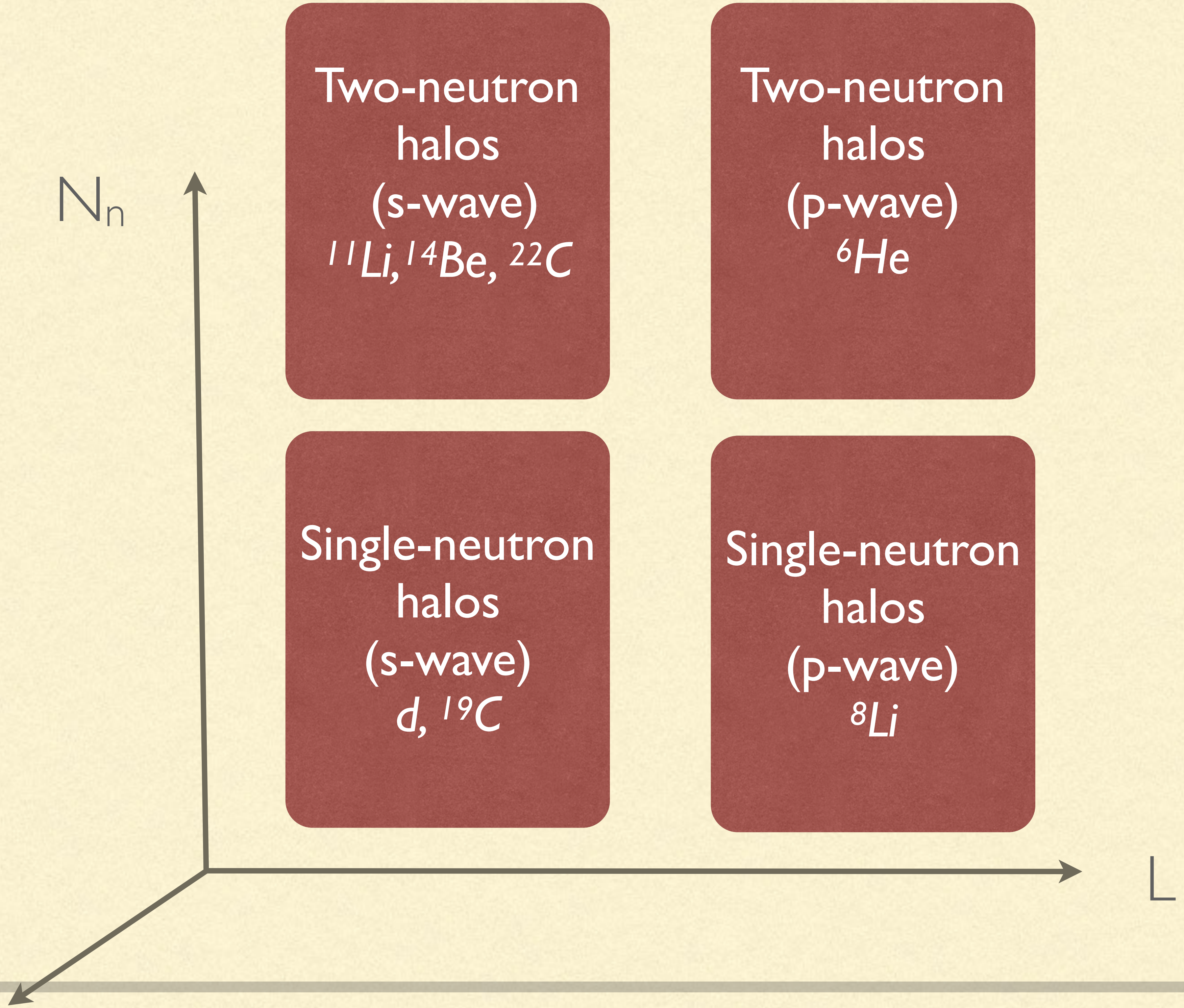
Single-neutron
halos
(s-wave)
d, ^{19}C

Single-neutron
halos
(p-wave)
^{8}Li

L

N_p

The multi-dimensional Halo EFT space



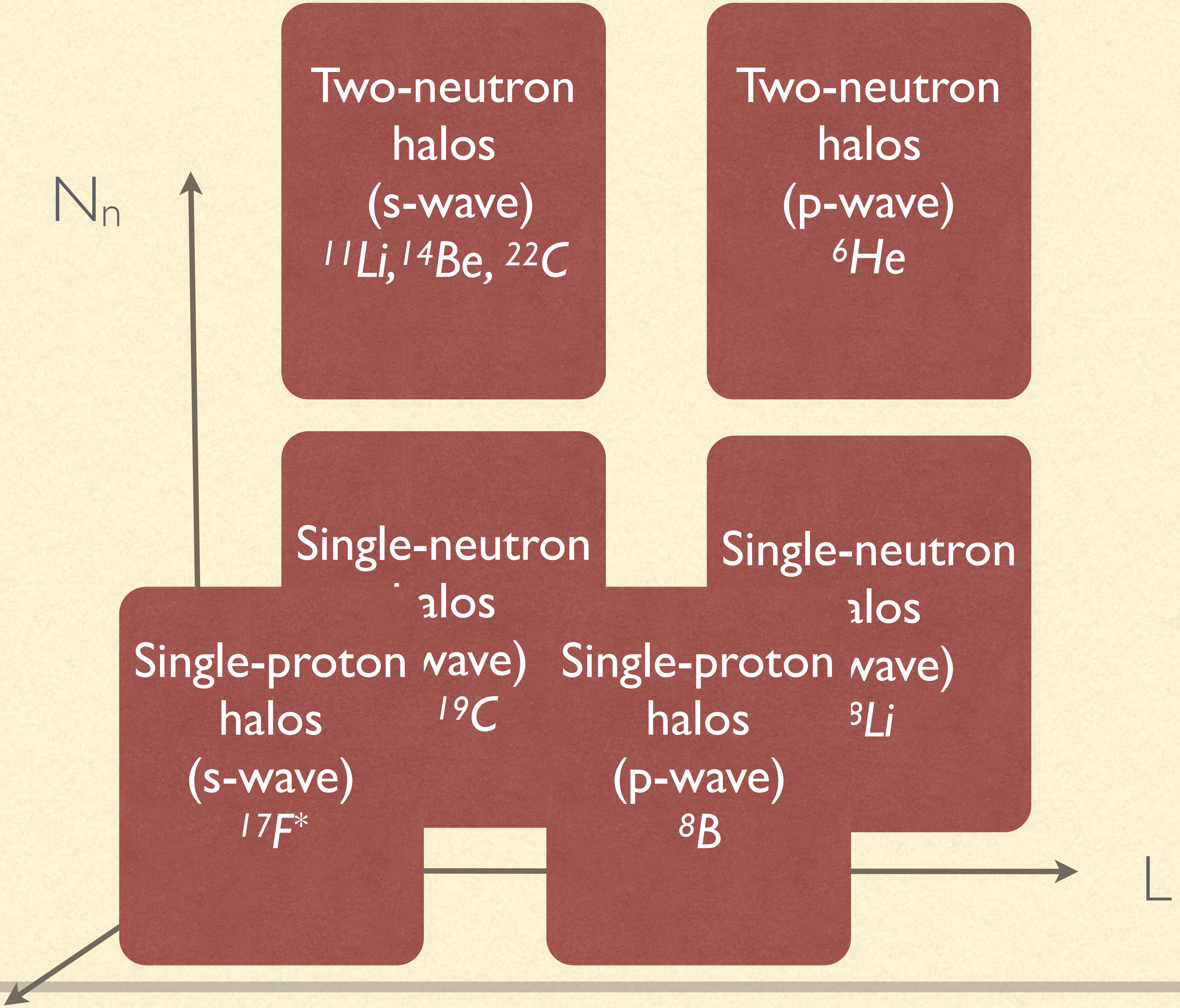
Two-neutron halos (s-wave)
^{11}Li, ^{14}Be, ^{22}C

Two-neutron halos (p-wave)
^{6}He

Single-neutron halos (s-wave)
d, ^{19}C

Single-neutron halos (p-wave)
^{8}Li

The multi-dimensional Halo EFT space



Two-neutron
halos
(s-wave)
$^{11}\mathit{Li}, ^{14}\mathit{Be}, ^{22}\mathit{C}$

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Single-proton
halos
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halos
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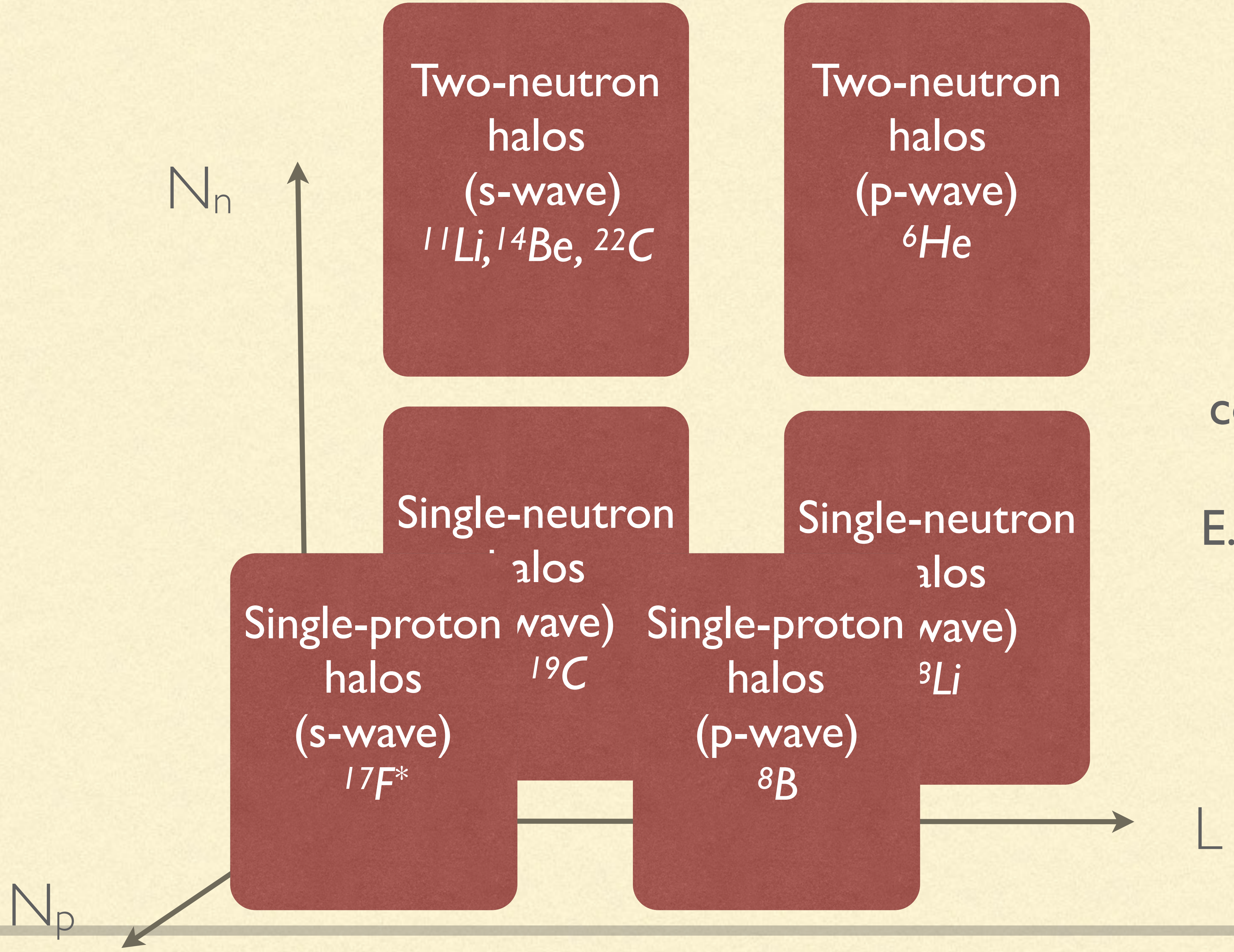

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alos
vave)
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halos
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Single-proton
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Plus
complementary
direction: N_α
E.g. ^{9}Be, $^{12}C^{*}$, etc.

Fixing ^{8}Li parameters

- ^{8}Li ground state is 2^{+}: both $^{5}P_{2}$ and $^{3}P_{2}$ components
- ^{8}Li first excited state: 1^{+}, bound by 1.05 MeV

Zhang, Nollett, Phillips, PRC (2014)

c.f. Rupak, Higa, PRL 106, 222501 (2011);
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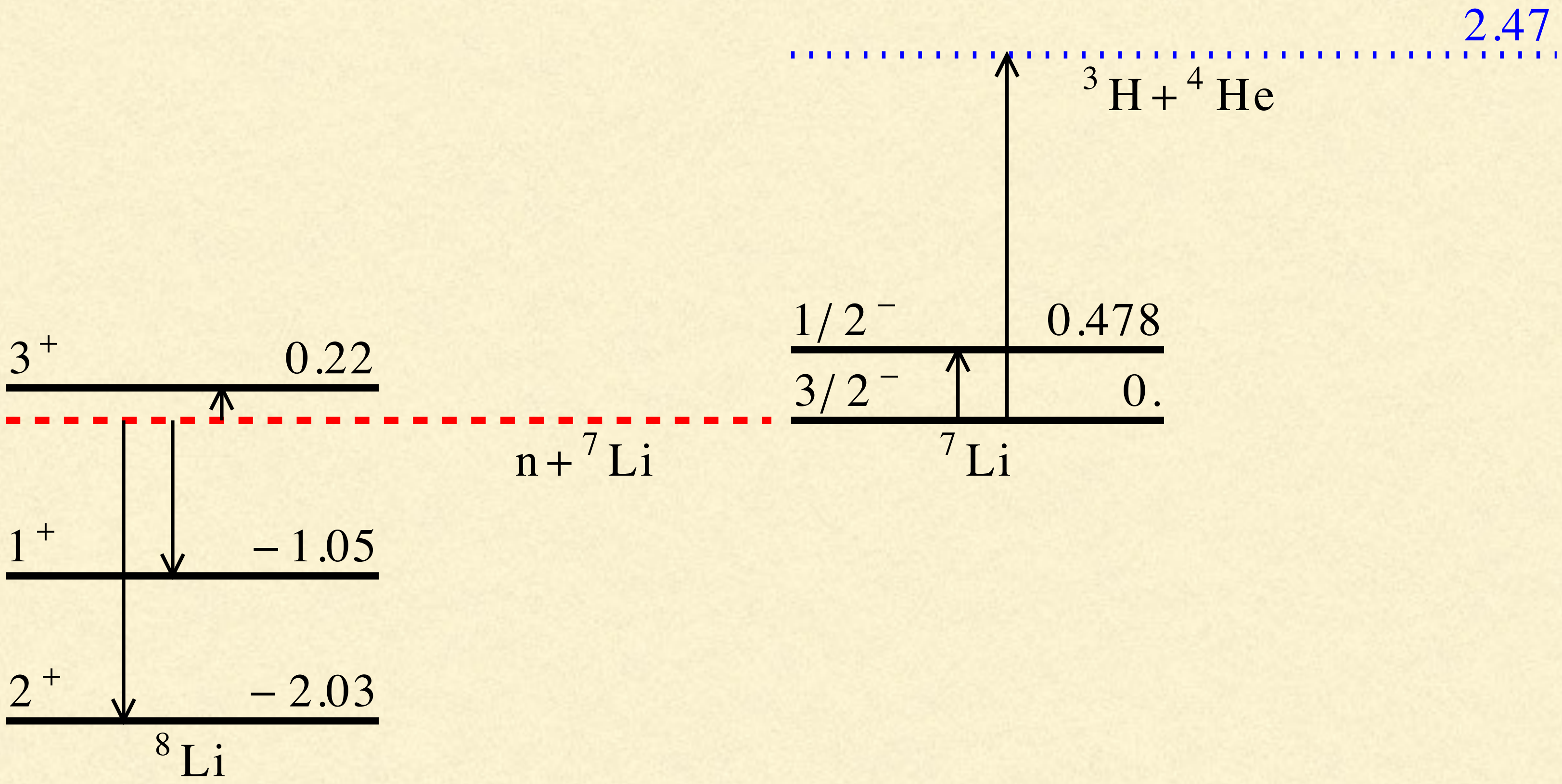

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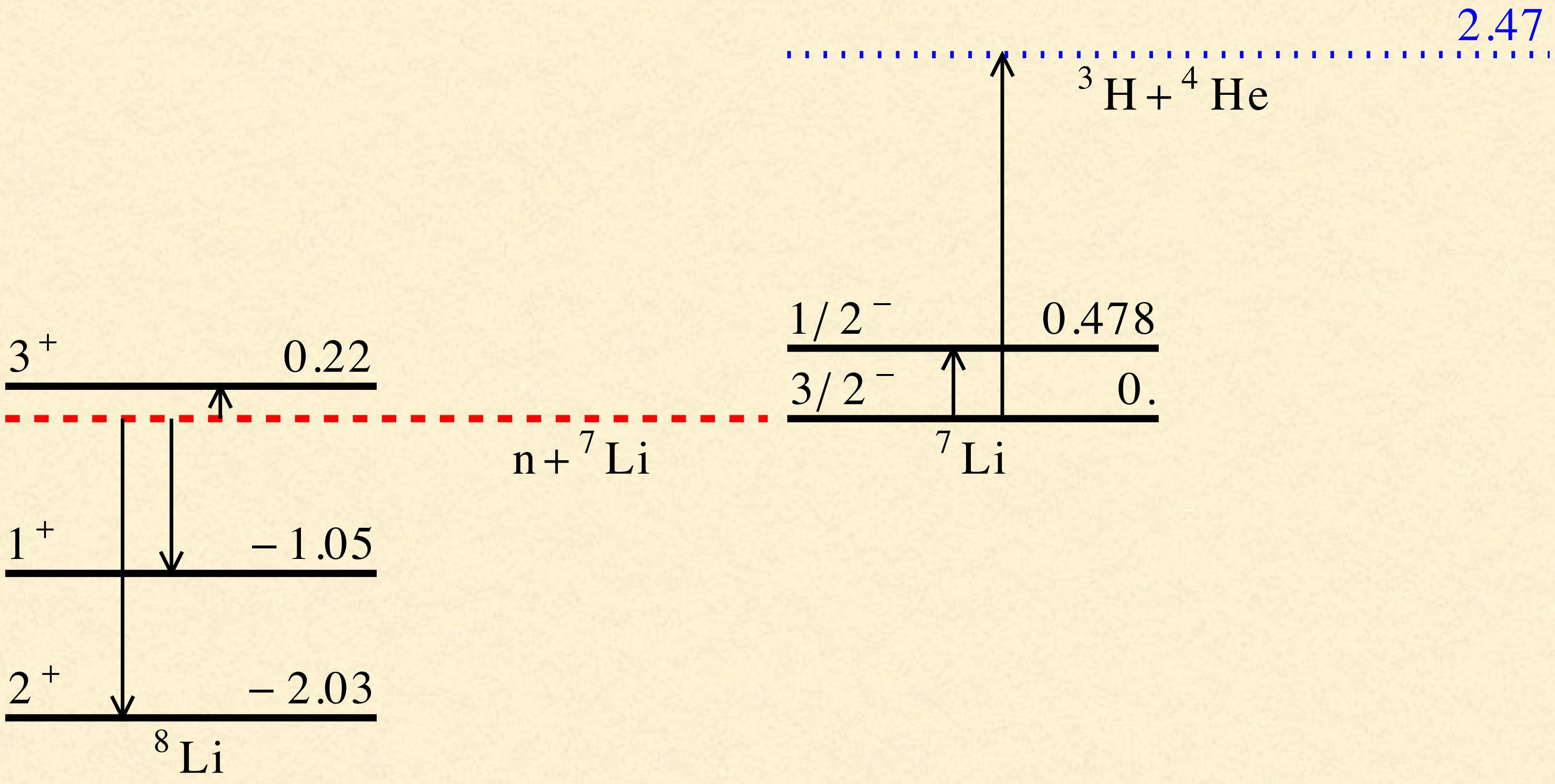

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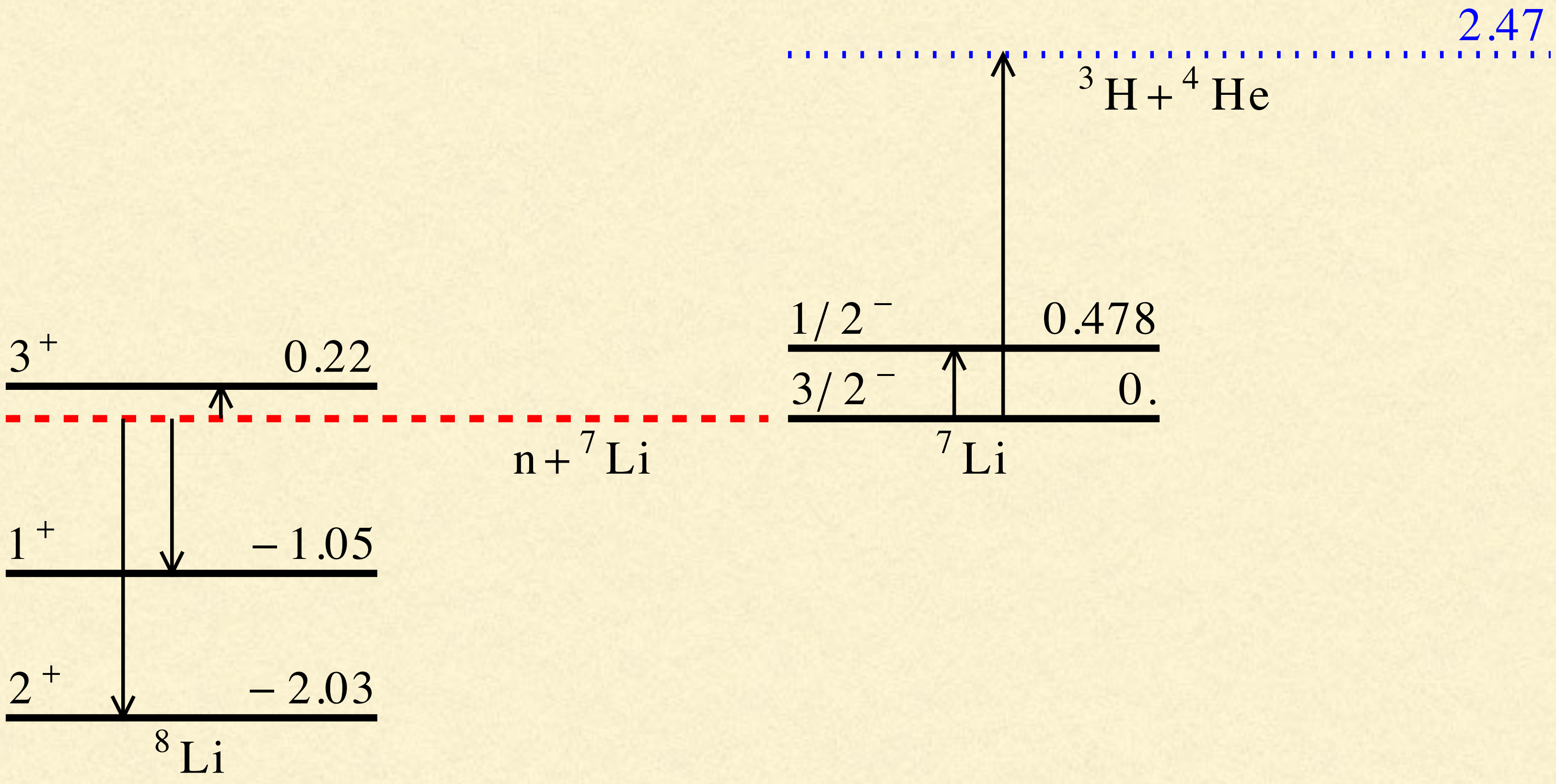

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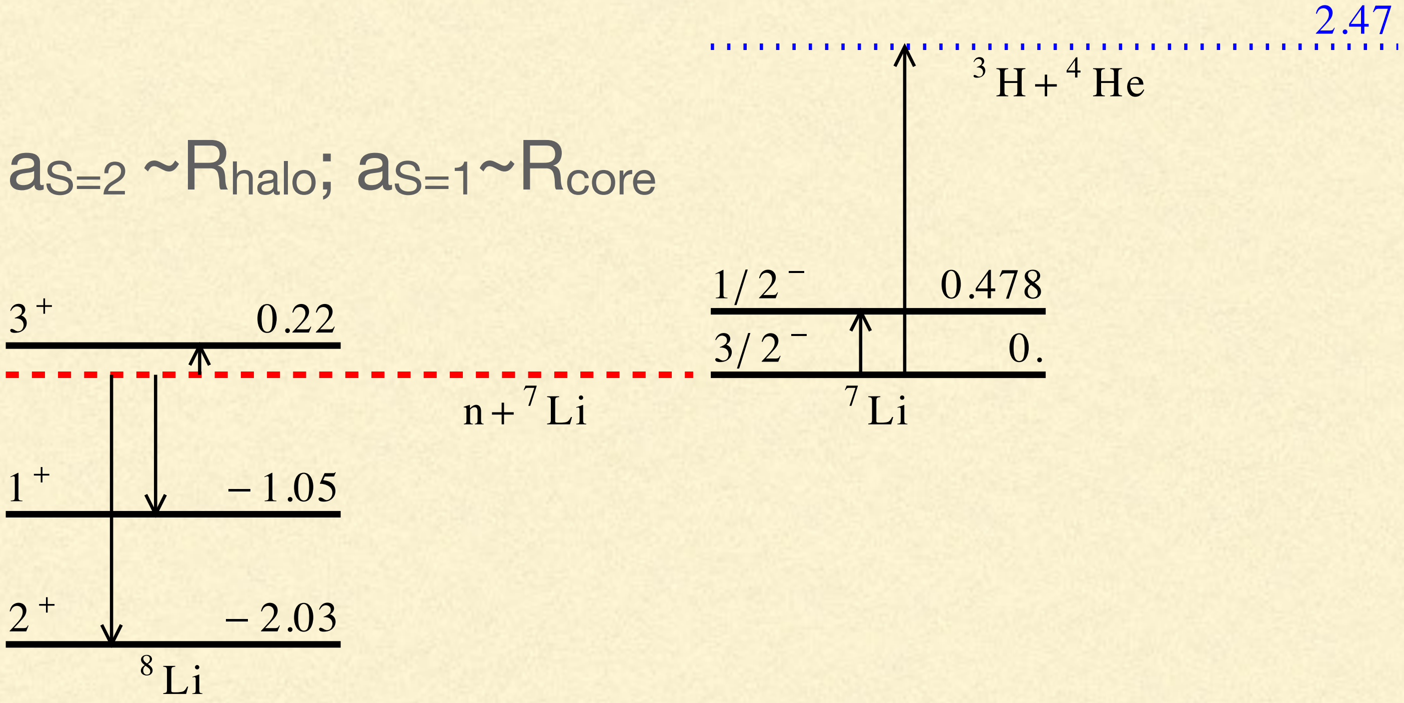
	$A_{(3P2)}$	$A_{(5P2)}$	$A_{(3P2^*)}$	$A_{(3P1)}^*$	$A_{(5P1)}^*$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)	0.220(6)	0.197(5)
Trache	-0.284(23)	-0.593(23)		0.187(16)	0.217(13)

^{7}Li + n$\rightarrow$^{8}Li + γ_{E1}

- ^{7}Li ground state is 3/2$^-$: S-wave n scattering in $^5S_2$ and $^3S_1$



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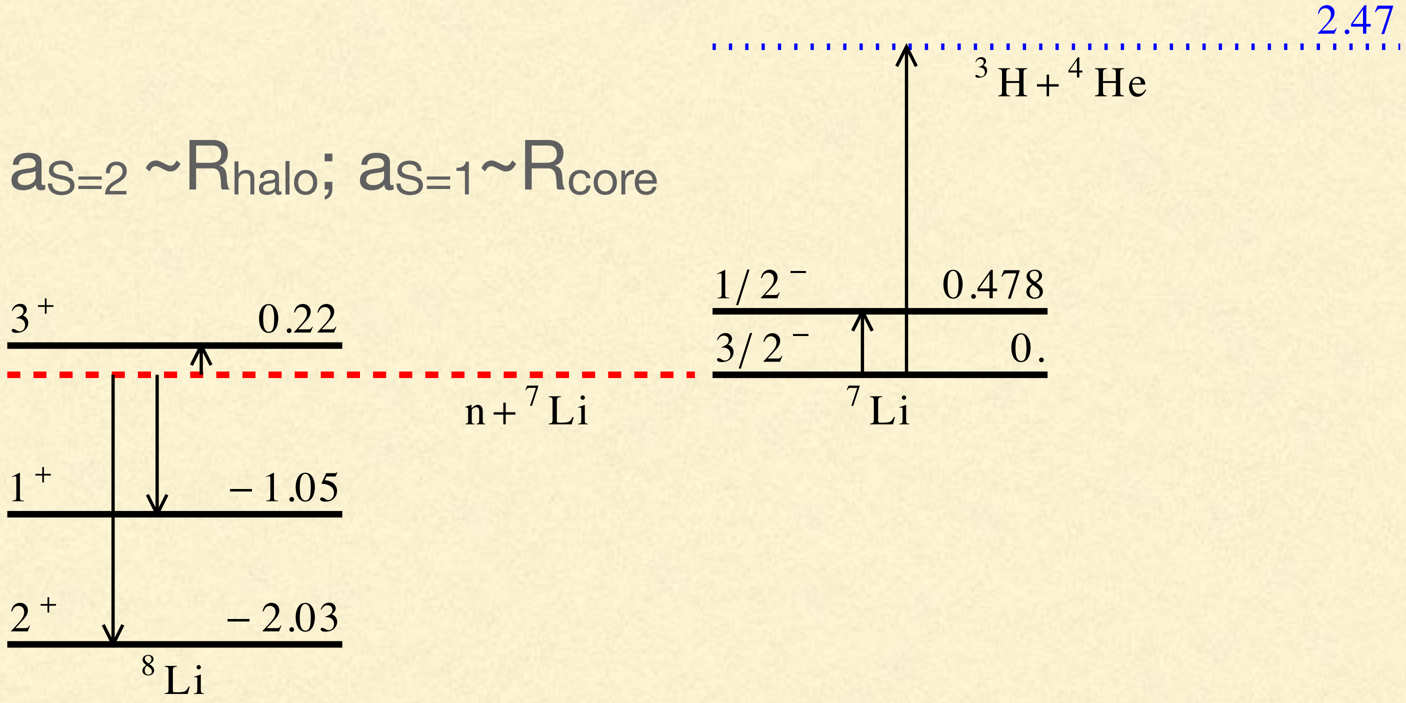

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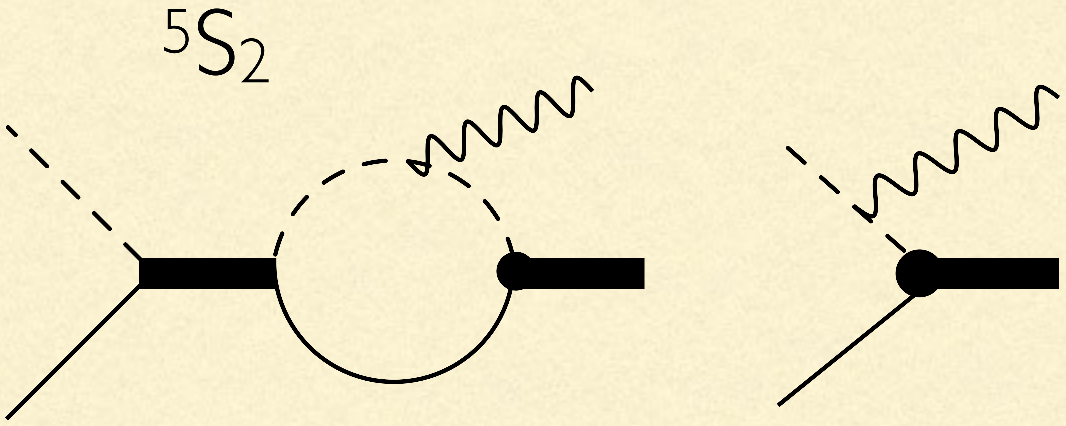

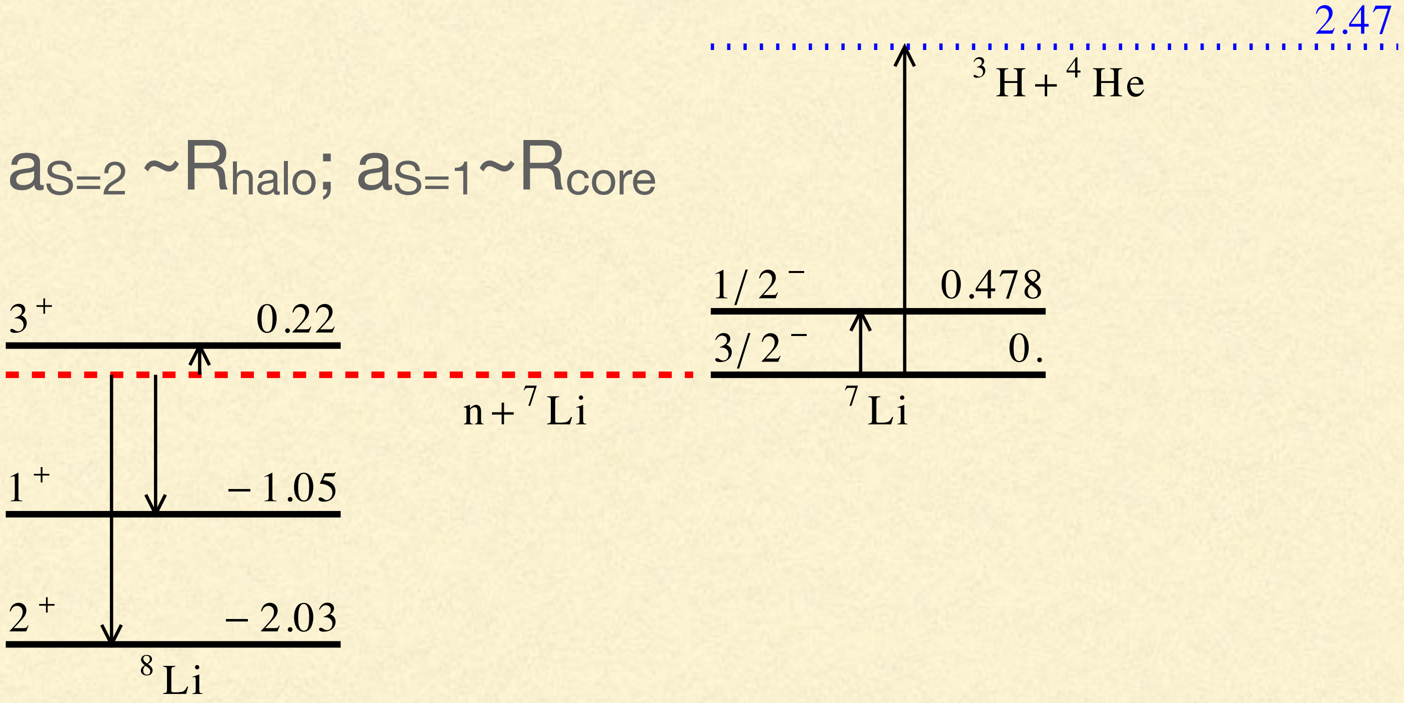

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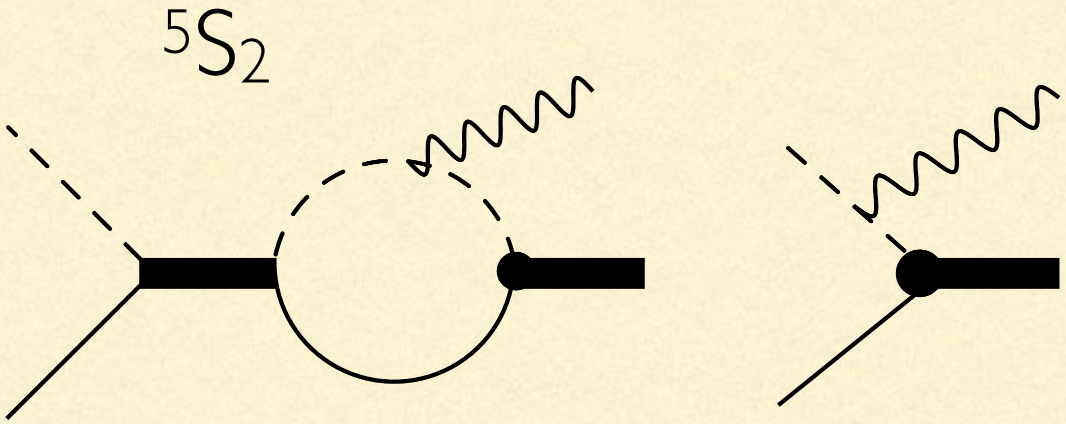

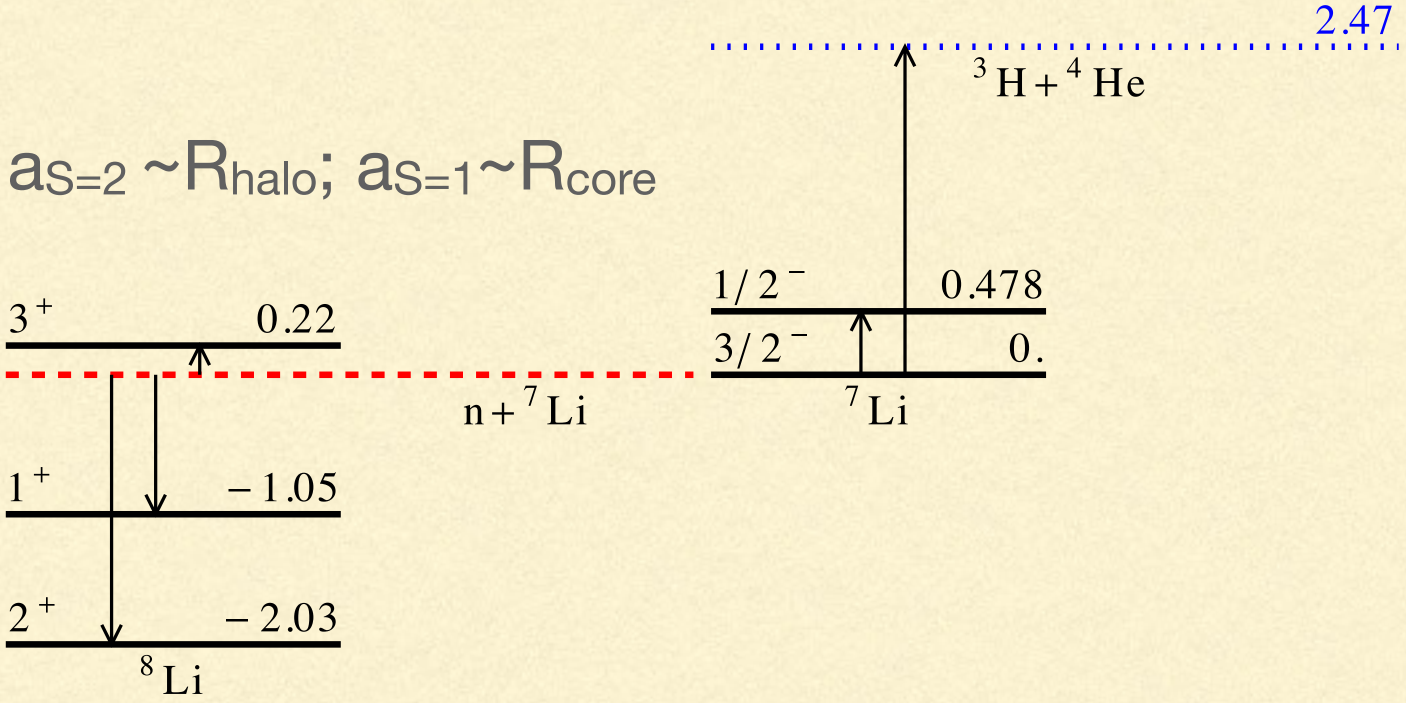

- LO calculation: S=2 (with ISI) and S=1 into P-wave bound state

$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r);$$

$$u_0(r) = 1 - \frac{r}{a}; u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

LO results for ^{7}Li + n→^{8}Li + γ_{E1}

Analysis: Zhang, Nollett, Phillips, PRC (2014)

Data: Barker (1996), cf. Nagai et al. (2005)

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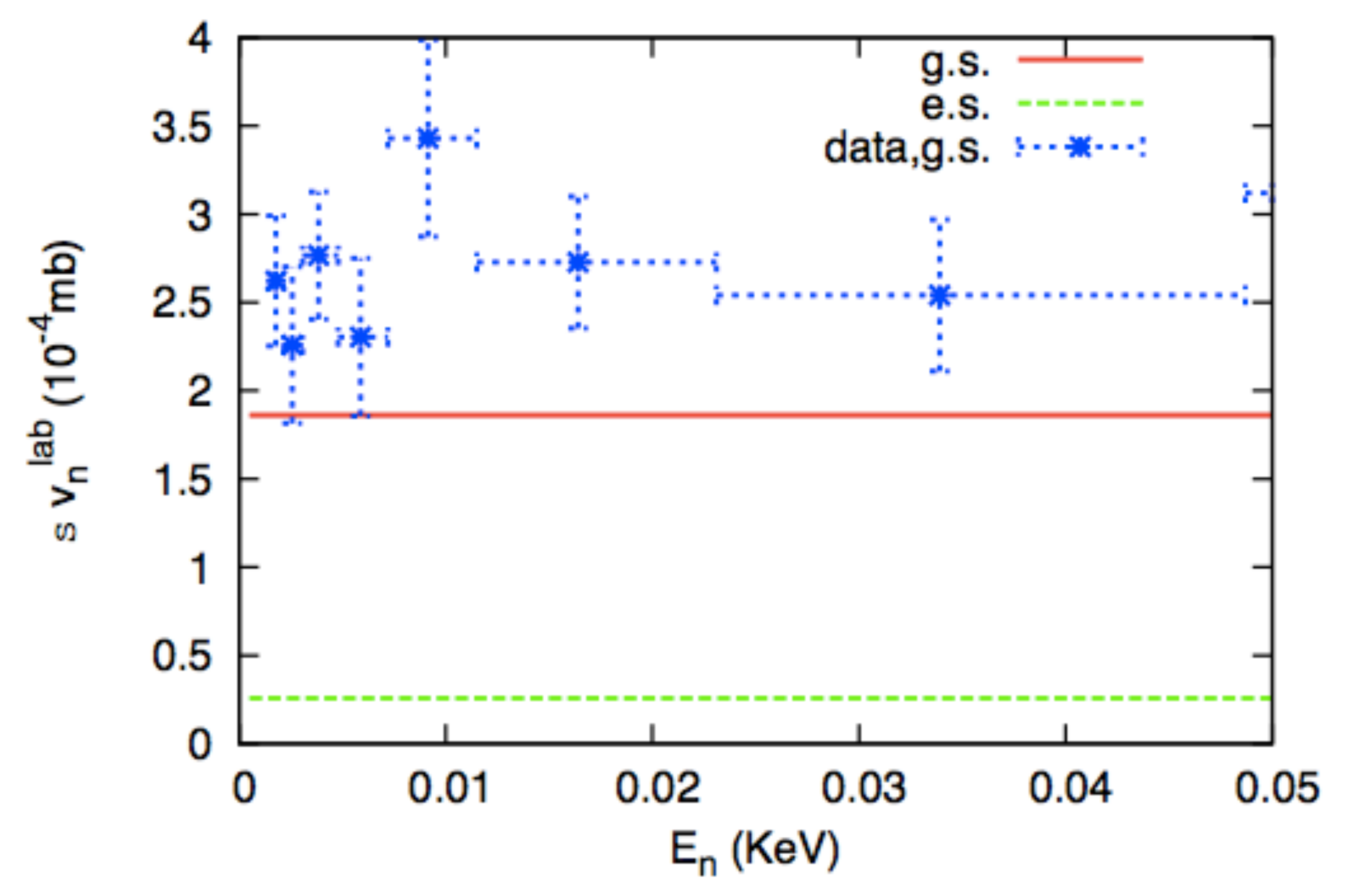

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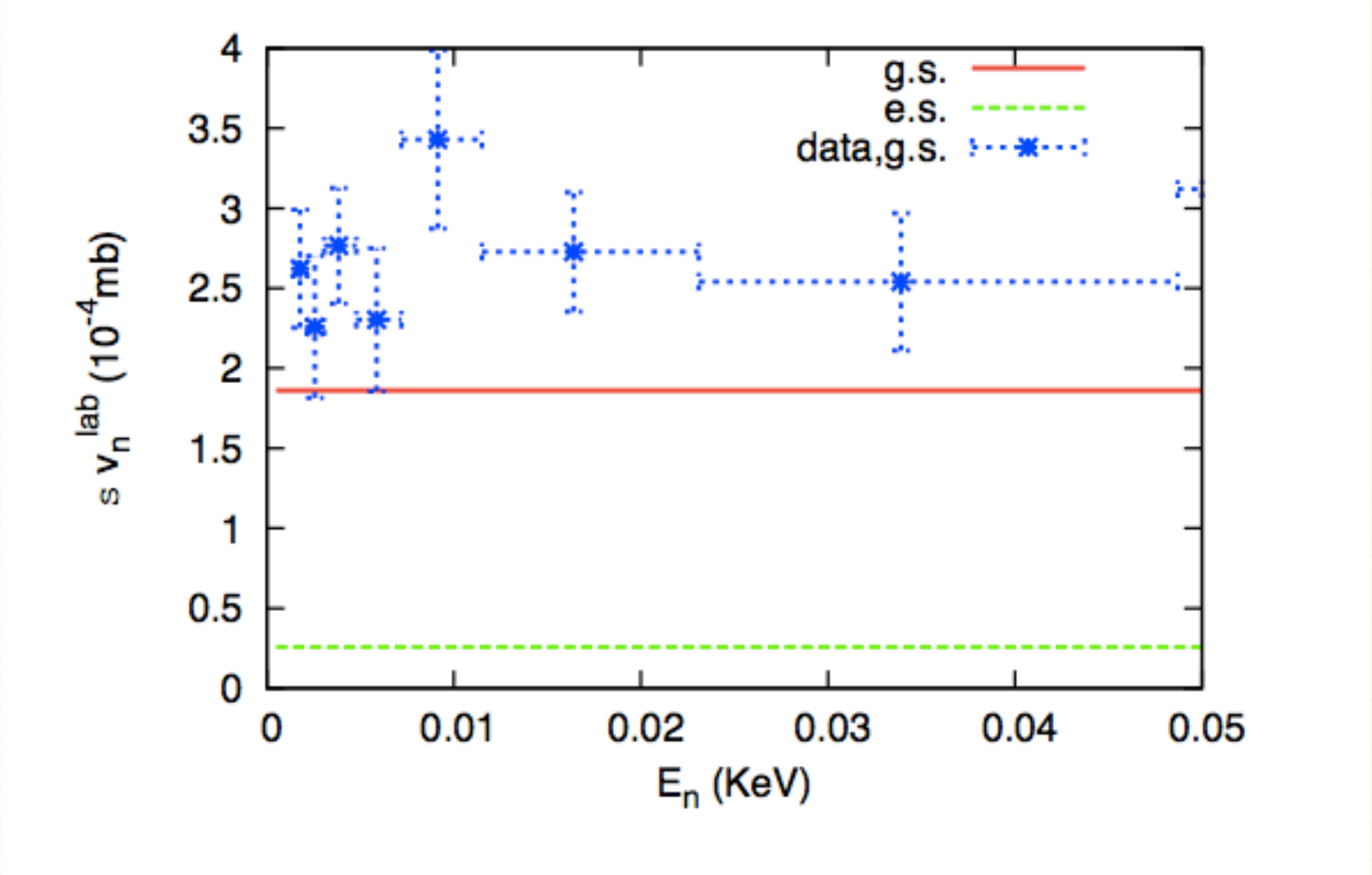

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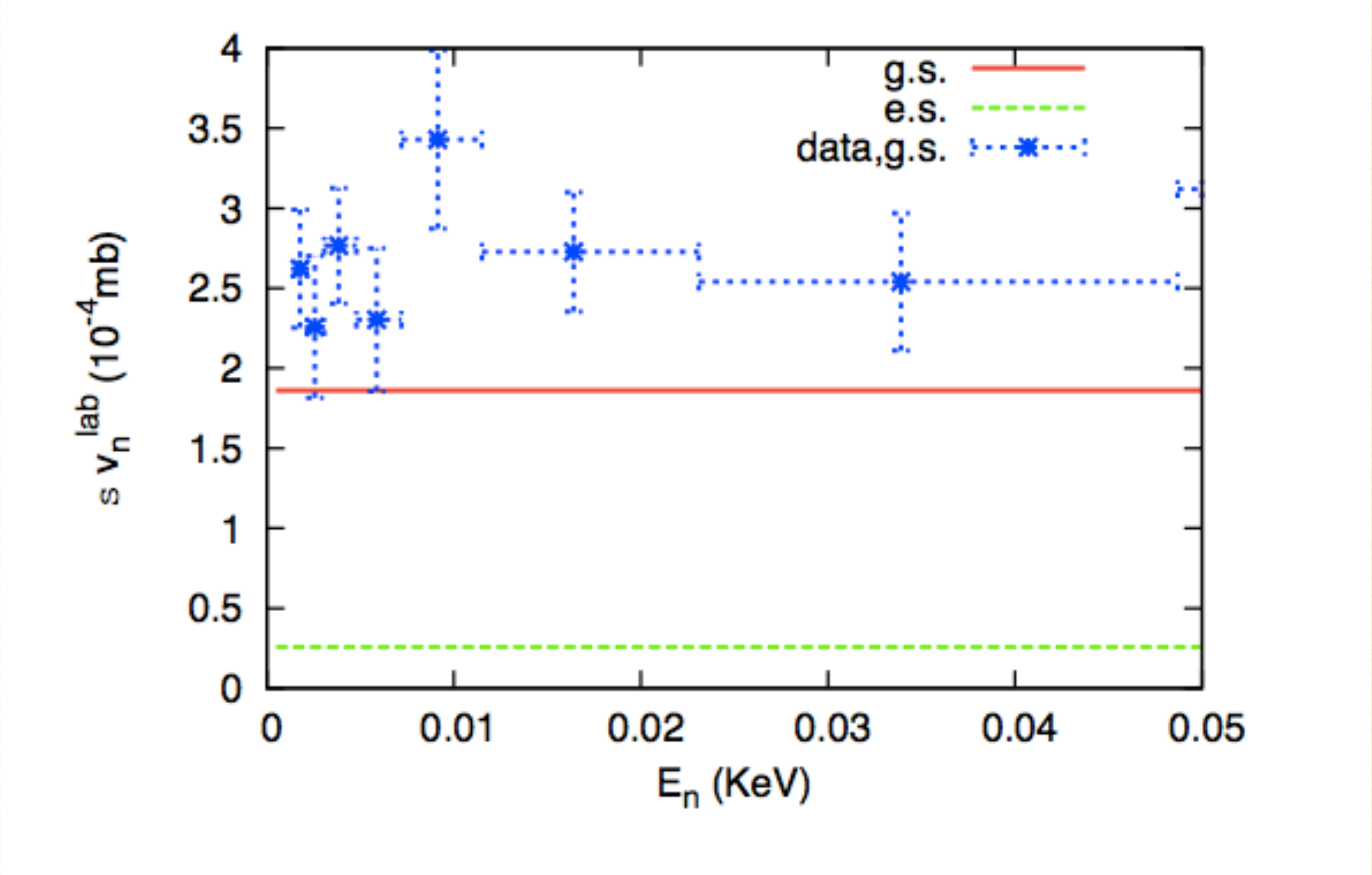

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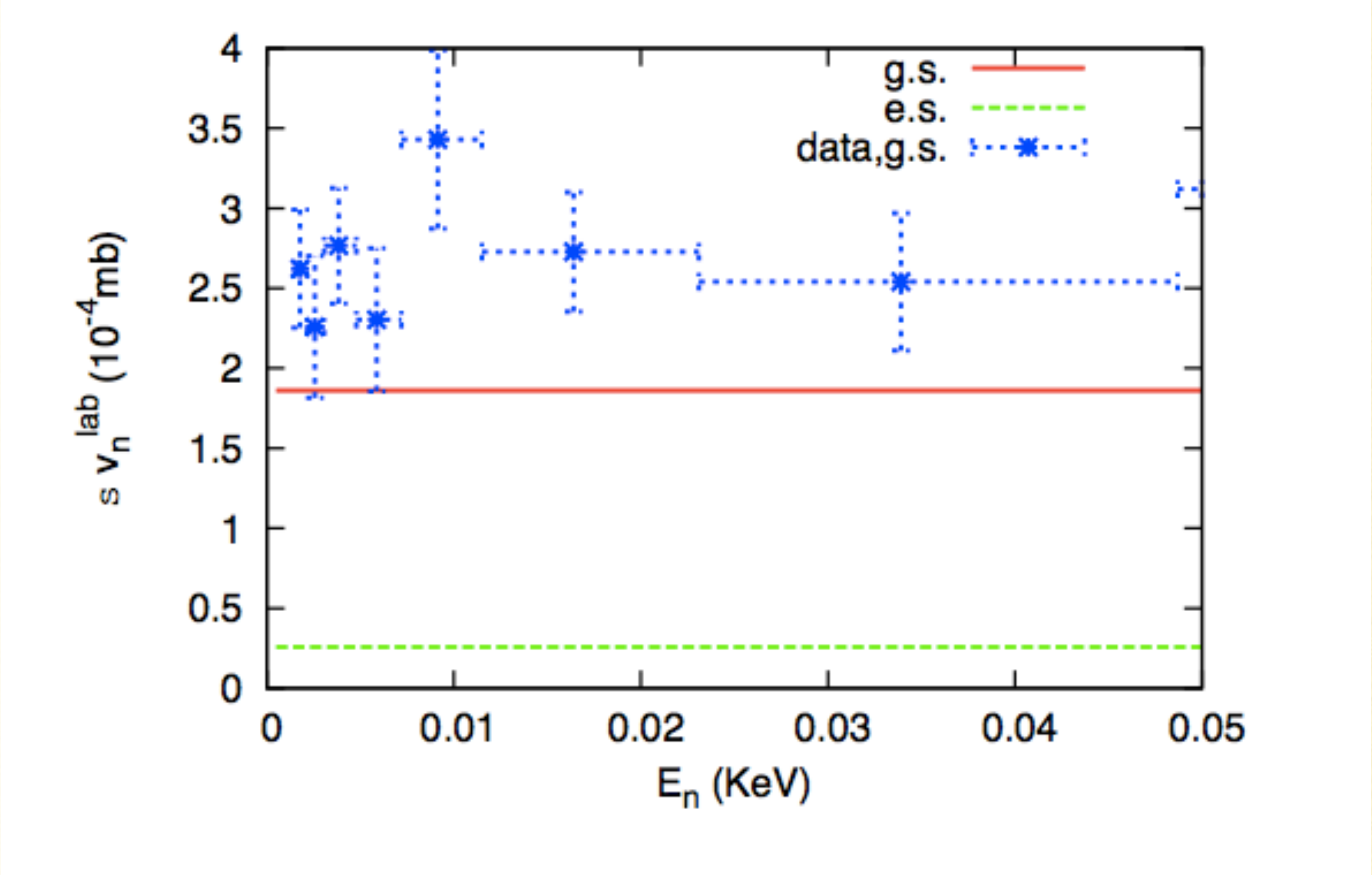

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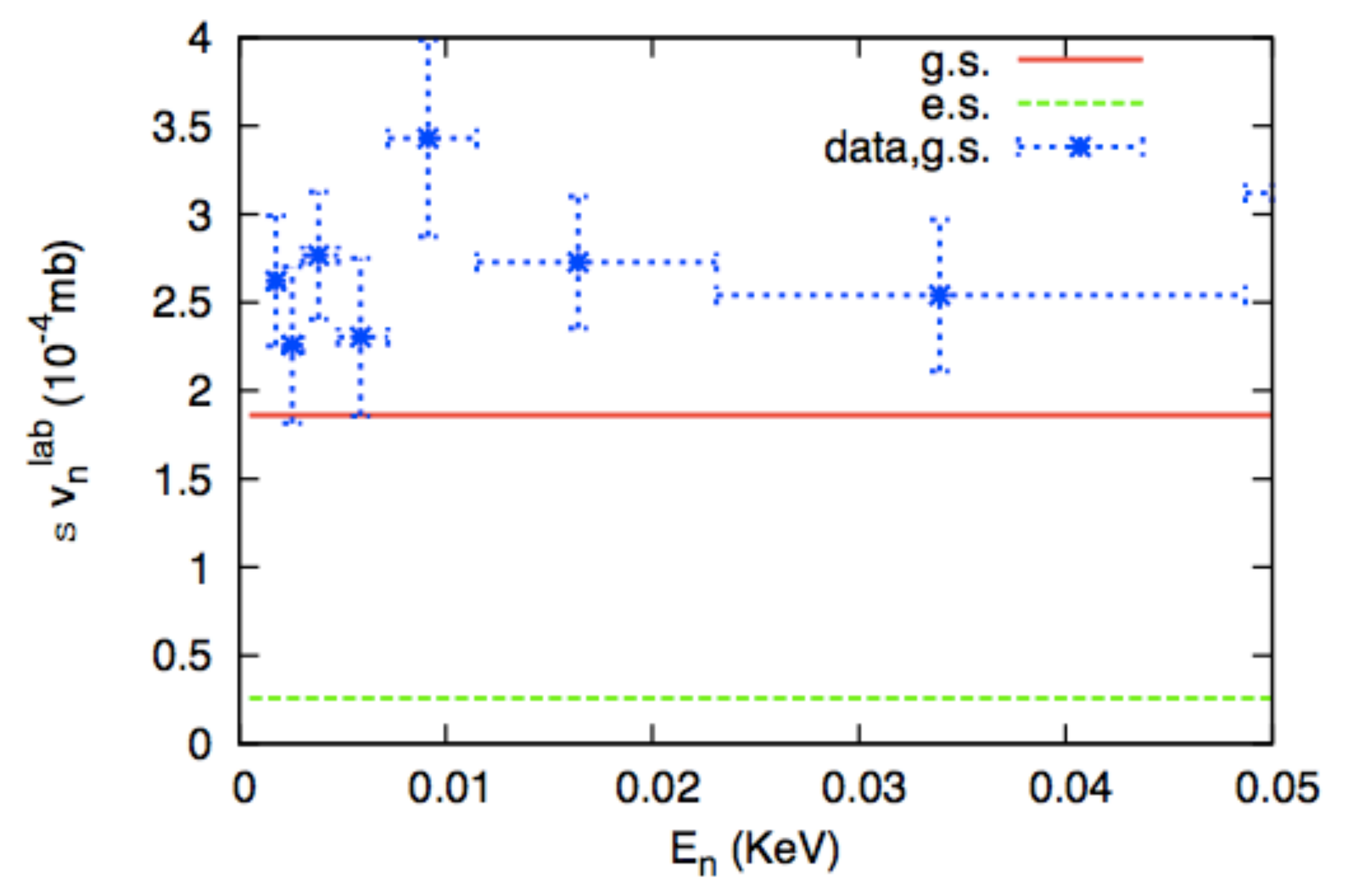

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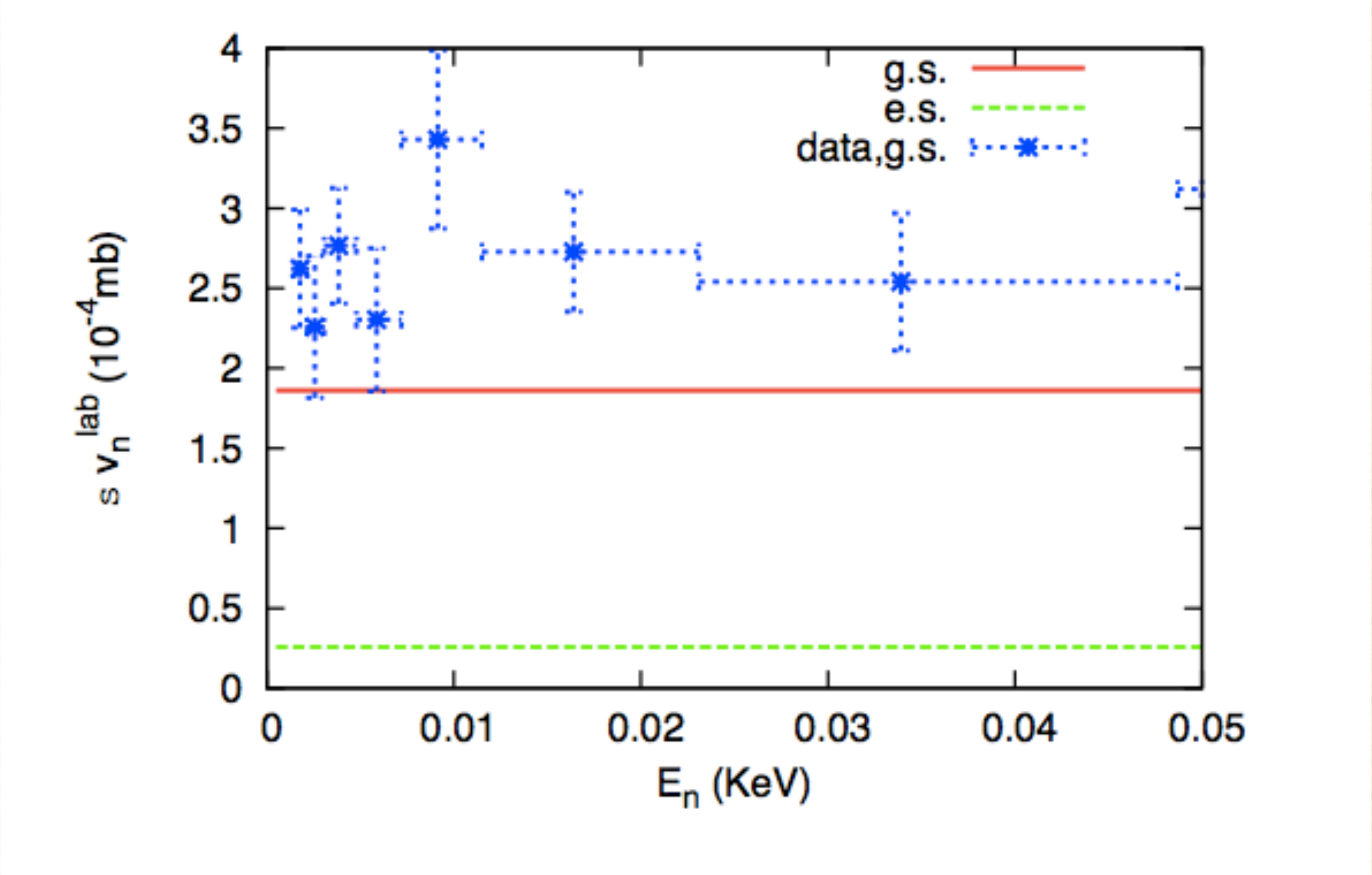

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Dynamics **predicted** through *ab initio* input

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

- Straightforward computation of diagrams yields:

$$\frac{d\mathrm{B(E1)}}{dE} = e^2 Z_{eff}^2 \frac{m_R}{2\pi^2} A_0^2 \left(\frac{p'^3[2p'^3\cot(\delta^{(1/2)}(p')) + \gamma_0^3 + 3\gamma_0 p'^2]^2}{[p'^6 + p'^6\cot^2(\delta^{(1/2)}(p'))](p'^2+\gamma_0^2)^4} + \frac{8p'^3}{(p'^2+\gamma_0^2)^4} \right)$$

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Spin-1/2 channel

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Wf renormalization

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- Higher-order corrections to phase shift at NNLO. Appearance of S-to-$^2P_{1/2}$ E1 counterterm also at that order.

Lagrangian: shallow S- and P-states

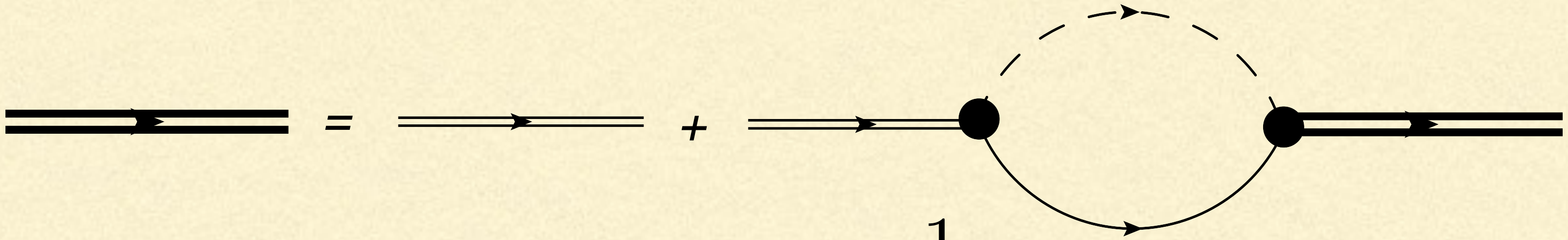
$$\begin{aligned}\mathcal{L} \quad = \quad & c^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)c + n^\dagger\left(i\partial_t + \frac{\nabla^2}{2m}\right)n \\ & +\sigma^\dagger\left[\eta_0\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_0\right]\sigma + \pi_j^\dagger\left[\eta_1\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_1\right]\pi_j \\ & -g_0\left[\sigma n^\dagger c^\dagger + \sigma^\dagger nc\right] - \frac{g_1}{2}\left[\pi_j^\dagger(n\; i\overleftrightarrow{\nabla}_j\; c) + (c^\dagger\; i\overleftrightarrow{\nabla}_j\; n^\dagger)\pi_j\right] \\ & -\frac{g_1}{2}\frac{M-m}{M_{nc}}\left[\pi_j^\dagger\; i\overrightarrow{\nabla}_j\; (nc) - i\overleftrightarrow{\nabla}_j\; (n^\dagger c^\dagger)\pi_j\right] + \ldots\,,\end{aligned}$$

- c, n: "core", "neutron" fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Dyson equation for (cn)-system propagator



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

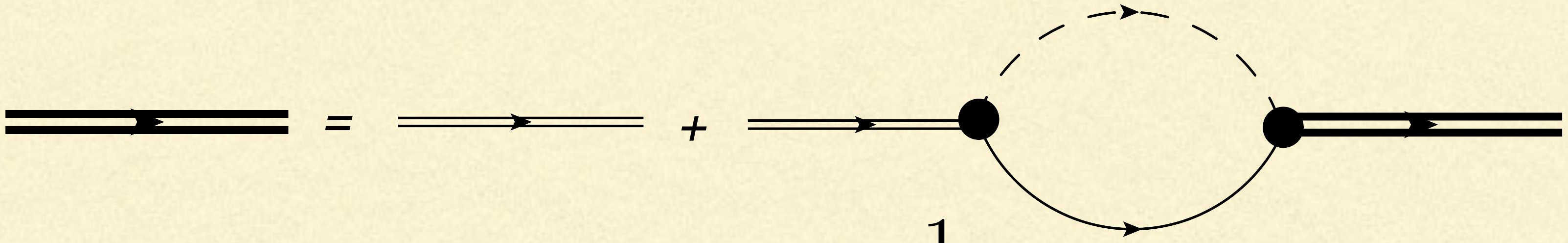
$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

- Reproduces ERE. But here (cf. s waves) cannot take r_1=0 at LO

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Dyson equation for (cn)-system propagator



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

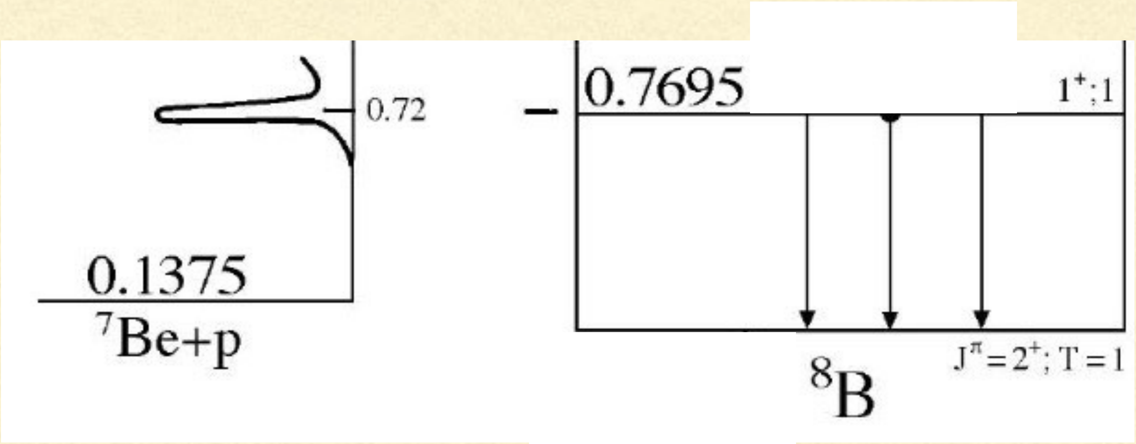
- Reproduces ERE. But here (cf. s waves) cannot take r_1=0 at LO

- If $a_1 > 0$ then pole is at $k=i\gamma_1$ with $B_1=\gamma_1^2/(2m_R)$:

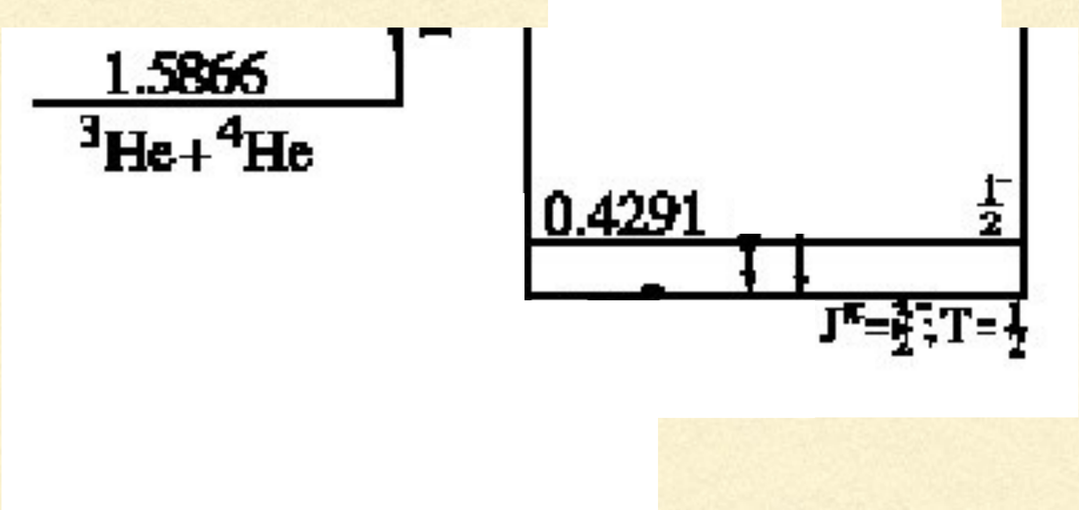
$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1+3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

Scales in the ^{8}B system

http://www.tunl.duke.edu



^{8}B



^{7}Be

Parameters for ^{7}Be + p→^{8}B + γ_{E1} at LO

Zhang, Nollett, Phillips, PRC (2014)
cf. Ryberg, Forssen, Hammer, Platter, EPJA (2014)

$$S(E) = f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\rm EC}\left(E; \delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

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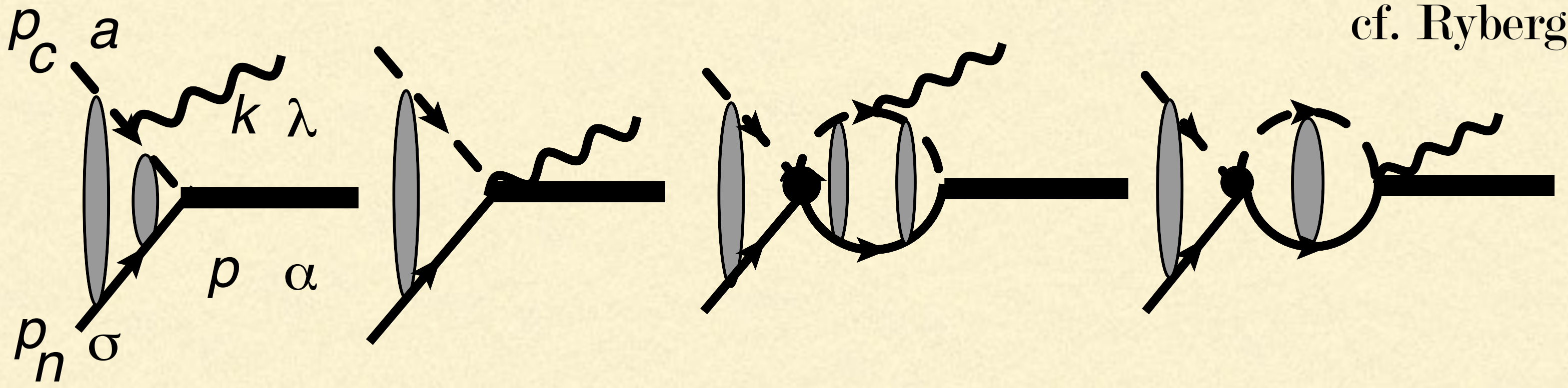

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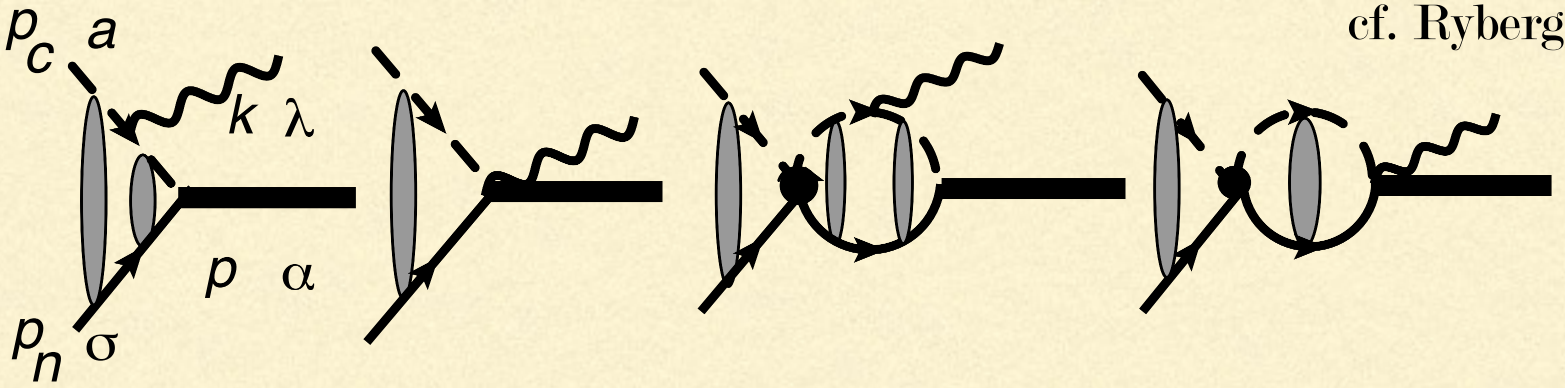

Four parameters at leading order

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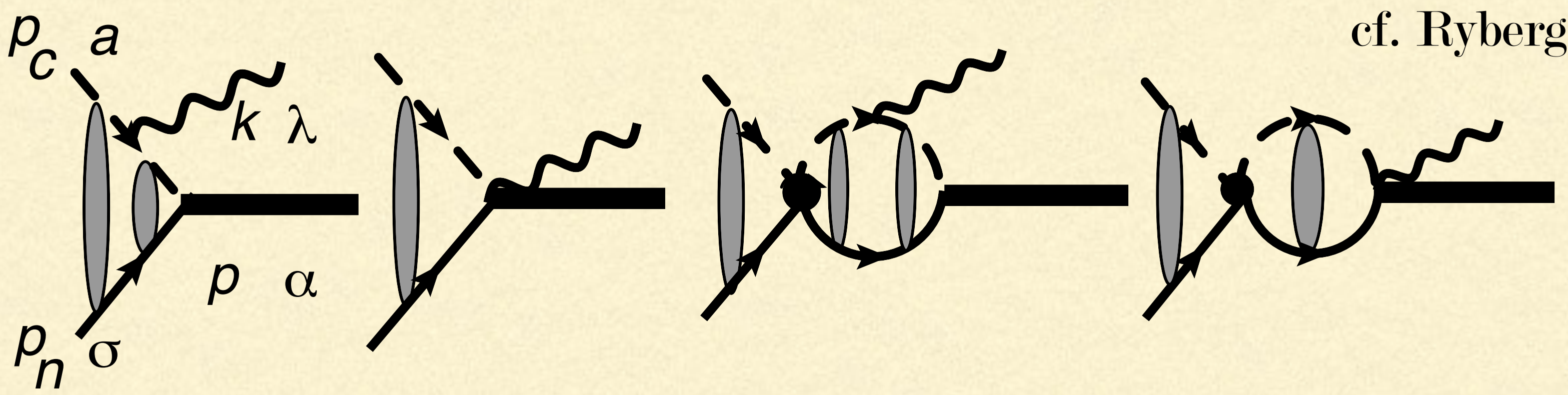

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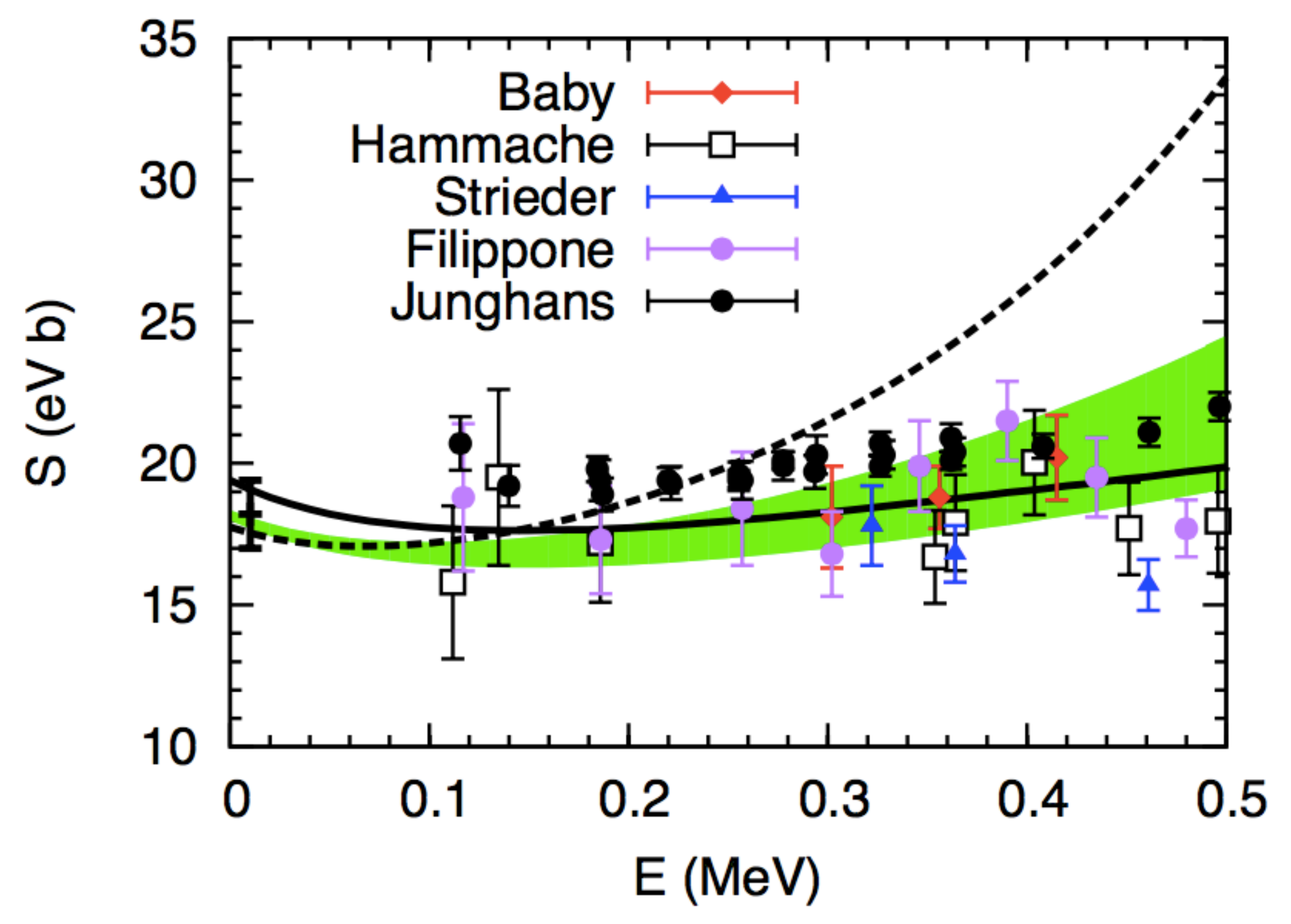
	$A_{(3P2)}$ (fm$^{-1/2}$)	$A_{(5P2)}$ (fm$^{-1/2}$)	$a_{(S=1)}$ (fm)	$a_{(S=2)}$ (fm)
Nollett	-0.315(19)	-0.662(19)		
Navratil et al.	-0.294	-0.65	-5.2	-15.3
Tabacaru	-0.294(45)	-0.615(45)		
Angulo			25(9)	-7(3)

Proton capture on ^{7}Be at LO: results

- ANCs yield r_1=-0.34 fm^{-1}, consistent with estimated scale Λ



Sensitivity to
input $a_{S=2}$ and $a_{S=1}$
at higher energies

At solar energies it's
all about the ANCs

Halo EFT as a “super model”

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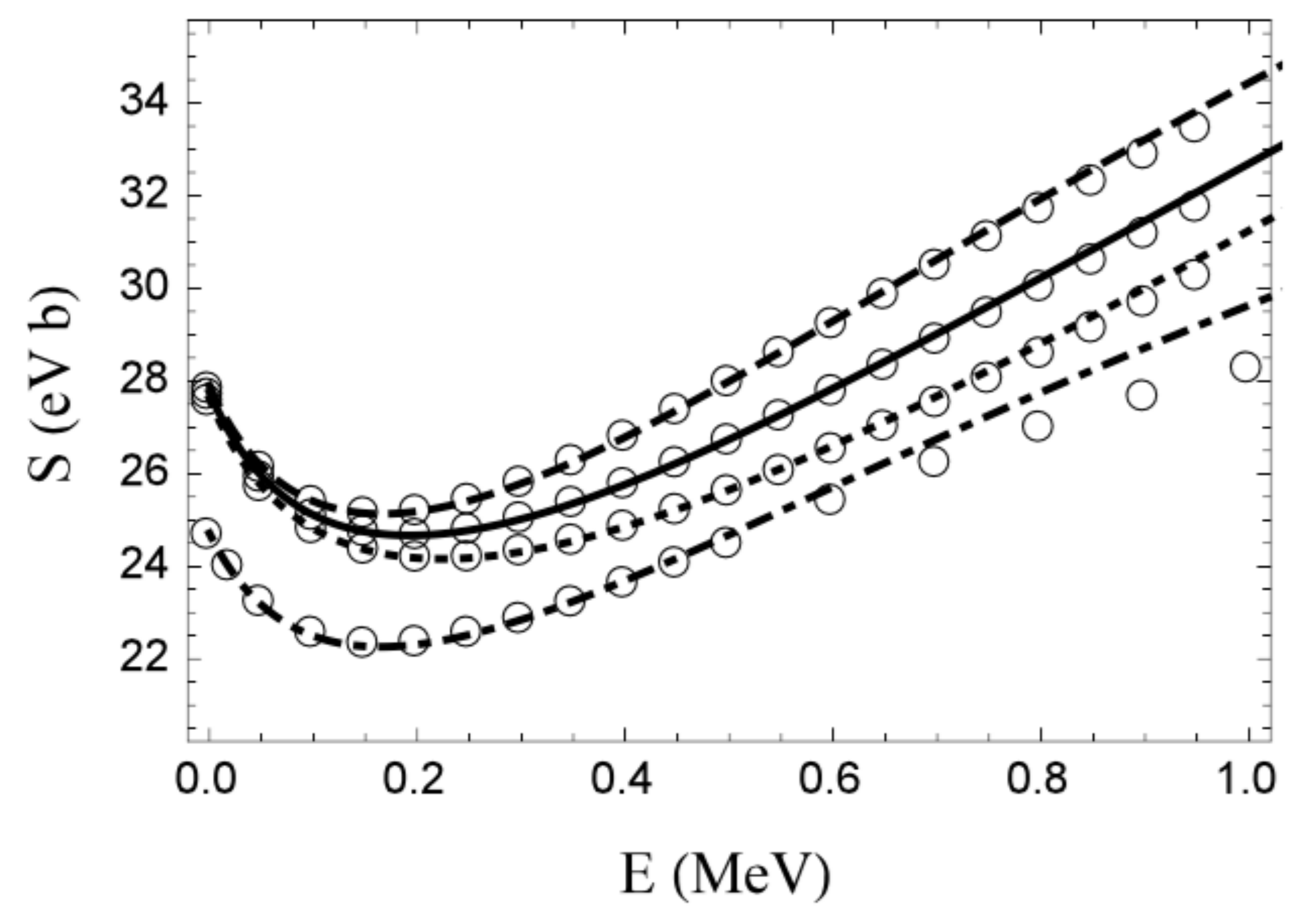

Halo EFT as a "super model"

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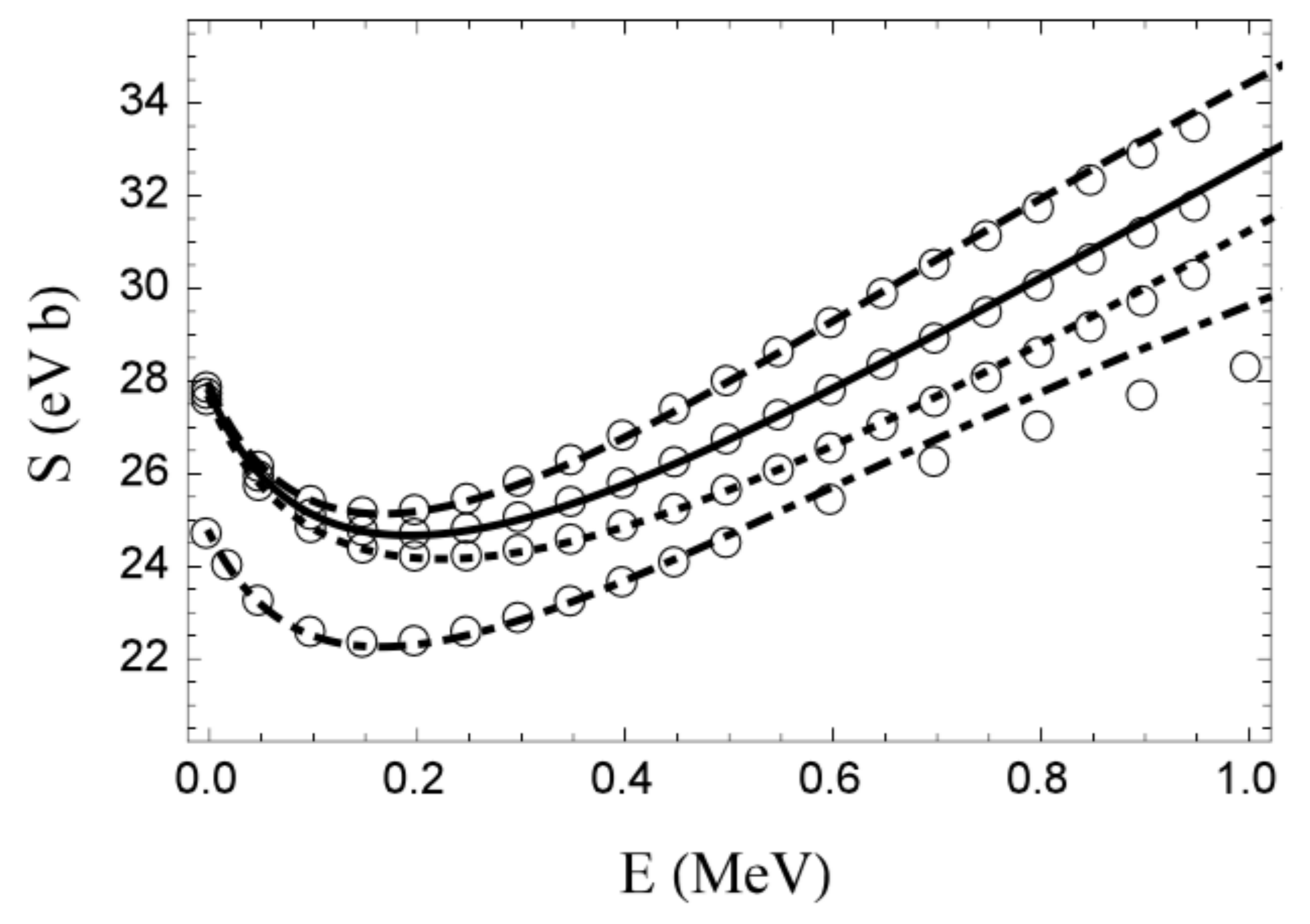
- Size of S(0) over-predicted in all models; curves rescaled in SFII fits



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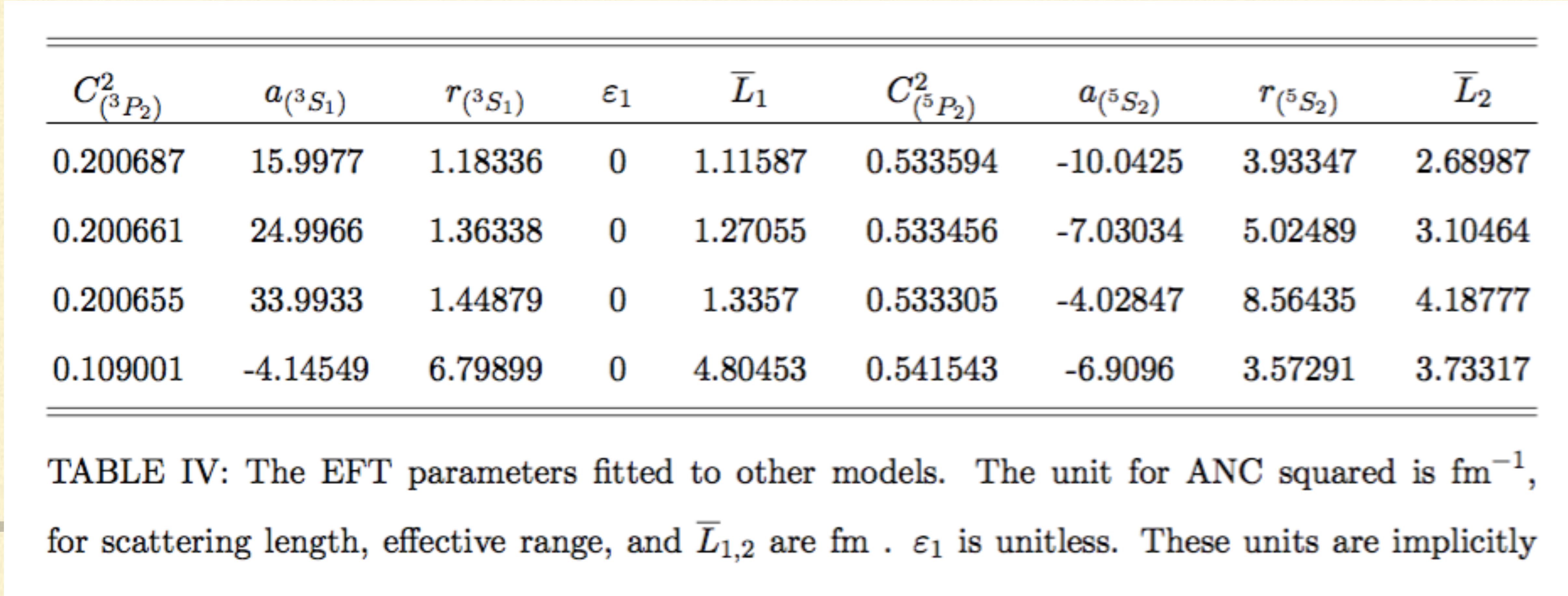
- Differences are sub-% level between 0 and 0.5 MeV
- Size of S(0) over-predicted in all models; curves rescaled in SFII fits
- Parameters generally obey a~1/R_{halo}, r ~R_{core}, L~R_{core}, as expected



$C^2_{(^3P_2)}$	$a_{(^3S_1)}$	$r_{(^3S_1)}$	ε_1	$\overline{L}_1$	$C^2_{(^5P_2)}$	$a_{(^5S_2)}$	$r_{(^5S_2)}$	$\overline{L}_2$
0.200687	15.9977	1.18336	0	1.11587	0.533594	-10.0425	3.93347	2.68987
0.200661	24.9966	1.36338	0	1.27055	0.533456	-7.03034	5.02489	3.10464
0.200655	33.9933	1.44879	0	1.3357	0.533305	-4.02847	8.56435	4.18777
0.109001	-4.14549	6.79899	0	4.80453	0.541543	-6.9096	3.57291	3.73317

TABLE IV: The EFT parameters fitted to other models. The unit for ANC squared is fm^{-1}, for scattering length, effective range, and $\overline{L}_{1,2}$ are fm . ε_1 is unitless. These units are implicitly

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- Taking a variety of "reasonable models" and using them to extrapolate may **over**estimate the model uncertainty